

## MODEL-BASED IDENTIFICATION OF MANUFACTURING PROCESSES OPERATIONAL DYNAMIC PARAMETERS

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### ABSTRACT

*The increasing demands for precision and efficiency of machining ask for development of new control strategies of a machining system based on the identification of its static and dynamic characteristics under operational conditions. This paper presents a procedure for formulating an analytic model of the dynamics of the machining system based on the identification of the system's parameters during its normal operation. This provides realistic prerequisites for in-process machining system testing. The models based on on-line identification may be used to control dynamic stability in machining and further for implementing a pro-active machining system optimization by correlating model parameters to for instance surface roughness features. This qualitative identification procedure and model parameters are used to formulate a decision rule for ascribing to a given machining process one of possible type of classes. The decision rule is formulated in terms of certain statistical characteristics in such a way to minimize the classification errors.*

**KEYWORDS:** machining, stability, ARMA, modelling, classification

## 1. INTRODUCTION

### 1.1. Problem definition

The primary task of process control in a manufacturing environment is to improve the flow of materials and the quality of parts. One of the limiting factors for achieving high accuracy and/or high material removing rates is posed by the dynamic effects brought about by the interaction between structural and process parameters during any machining operations. The problem of machine tool vibration has been thoroughly studied and is well documented in literature [9, 11]. The issue is periodically revived by the steady introduction of new structural materials as well as the attempt to achieve a higher efficiency of manufacturing processes in a dynamic and harsh manufacturing environment. Model-based identification for stability analysis and chatter control is treated in [2, 4, 7].

In milling, the time-varying and discontinuous nature of the machining system represents a challenge from the point of view of parameter selection, control and optimization. Many of the methods used to analyze, control and optimize machining systems are based on off-line procedures or test environments that do not exactly replicate the actual machining

operation [13]. The fundamental problem in the stability of a machining system is the discrimination between forced vibrations and self-excited vibration, which is treated in this paper in view of the following considerations:

1. Formulation of a qualitative/semi-qualitative mathematical model of the machining system for subsequent quantitative analysis.
2. Evaluation of the system's stability boundary.
3. Implementation of a suitable design for real time monitoring and control.

The term 'qualitative' implies that the model is based on the statistical analysis of the measured system's response. Although the model-based identification approach presented in this paper leads to the estimation of key dynamic parameters, these parameters are nevertheless meaningful only within certain confidence intervals. The primary contribution of this paper lies within the formulation and implementation of parametric stochastic models for the study of the dynamic interaction between the machine tool structure and the chip formation in milling of hard materials. A criterion for the computation of the dissimilarity between various inferred models is formulated based on characteristic frequencies and the overall damping ratios (join

system, elastic structure-cutting process). Recursive model identification represents an advanced approach for real time monitoring, control and optimization of machining systems. The parametric stochastic models are discussed in section 2. Experimental procedure and results are presented in section 3. Discussions and conclusions are outlined in section 4 and 5, respectively.

## 2. CONCEPT OF MODEL-BASED IDENTIFICATION

The term identification refers to the formulation of a mathematical model of a dynamic system based upon signal measurements [6] and belongs to a class of inverse dynamic problems [10] encountered in various fields of natural science and technology. The key concept of the identification procedure in this paper is to find a feature of the measured random response that can be used to discriminate between machining systems of various types. This is a second order qualitative identification since no provision for quantitative estimation is done on the measured signal (as opposed to non-parametric identification methods).

First the model's parameters are estimated. The frequency and overall damping ratio (join system, elastic structure-cutting process) are then statistically computed from the model parameters and used as discrimination features [3]. The desired mathematical model of the machining system is based on the data obtained during normal operational conditions. In this way we take a step beyond the classical method of analyzing the dynamics of a machining system, which separately identifies the structural and process parameters. This type of identification and parametric modelling relies strongly on statistical methods because of the random nature of the cutting process. The novelty of this concept is represented by the extension of the model-based identification from off-line to recursive (sequential) parameter identification.

### 2.1. Parametric ARMA models

Parametric models used here are based on stochastic processes and a special class within this family is defined by autoregressive moving average or ARMA models [5, 8]. ARMA models offer an acceptable trade-off between flexibility and parsimony with respect to the number of model parameters. The model for an ARMA process can be expressed as

$$Y(z) = H(z)U(z) \quad (1)$$

where  $Y(z)$ ,  $U(z)$  and  $H(z)$  are the  $z$ -transforms of the output sequence, input sequence and the system impulse response (transfer function), respectively, and

$$H(z) = \frac{c_0 + c_1z^{-1} + c_2z^{-2} + \dots + c_qz^{-q}}{1 - a_1z^{-1} - a_2z^{-2} - \dots - a_pz^{-p}} \quad (2)$$

The input excitation in an ARMA process is not observable but can be assumed to be random and broadband compared with the measured output sequence. This is a true assumption for the machining system where machine tools are usually rather low damped structures interacting with the chip formation process. Dynamic variations resulting from the interaction between the dynamics of mechanical structures and the chip flow formation process may be considered to be broadband frequency excitation. Furthermore, in milling, the intermittent engagement of multi tooth cutters excites the structure with forces similar to impulses.

### 2.2. Physical parameter identification

For a second order under damped system with impulse response function given by

$$h(z) = Ae^{-\xi t} \sin(\omega t + \phi) \quad (3)$$

the  $c_i$  and  $a_i$  parameters of an ARMA model in equation (2) can be related to physical parameters,  $A$  (amplitude),  $\xi$  (damping) and  $\omega$  (angular frequency)

$$H(z) = \frac{A \sin \phi + Ae^{-\xi \Delta t} \sin(\omega \Delta t - \phi)z^{-1}}{1 - 2e^{-\xi \Delta t} \cos(\omega \Delta t)z^{-1} + e^{-2\xi \Delta t} z^{-2}} \quad (4)$$

where  $\Delta t$  is the sampling interval.

Estimation of the Power Spectral Density (PSD) of sampled data containing stochastic components is traditionally performed by the help of Fast Fourier Transform (FFT). There are however, a number of problems related to spectral analysis based on non-parametrical methods such as Fourier approach. The major limitation of FFT spectral analysis is the lack of ability to discriminate the spectral components of two signals. This becomes a major problem when attempting to analyze short time series because the frequency resolution is the inverse of the number of available samples. The classical PSD based on Discrete Fourier Transform (DFT) is given by

$$\Phi_{yy} = \left| \frac{1}{N} \sum_{n=0}^{N-1} y_n e^{-j2\pi n/N} \right|^2 \quad (5)$$

where  $N$  is the number of samples. The power spectrum for a process described by equation (5) is obtained by evaluating the impulse response function around the unit circle in  $z$ -transform plane,  $z^{-1} = \exp(-j2\pi fDt)$ .

Using only AR parameters, the PSD function is determined as follows

$$\phi_{AR} = \frac{\sigma^2 \Delta t}{\left| 1 + \sum_{n=1}^m a_n e^{-j2\pi n \Delta t} \right|^2} \quad (6)$$

The PSD can therefore be determined solely from the knowledge of the coefficients  $a_1, a_2, \dots, a_p$ , and the variance,  $\sigma^2$ .

In the model-based identification procedure, the estimation of physical parameters, operational frequencies  $\omega_{op}$  and operational damping ratios  $\xi_{op}$  can be used for the control of dynamic stability. By operational dynamic parameters we denote the combination of the structural vibration modes and process vibration modes resulting during machining system operation. It is important to stress that in the context of stochastic modelling, the estimated physical parameters are meaningful only from a statistical point of view, i.e. they are properly significant within a certain confidence interval. The mapping process from ARMA parameter domain to the  $\omega_{op} - \xi_{op}$  domain gives the advantage of robust chatter identification criteria. Theoretically, dynamic stability can be defined in terms of positive damping. A system is dynamically stable if the damping is positive and unstable when damping becomes negative. In machining, as we are interested in avoiding instabilities like chatter, when damping start to decrease towards zero it is a proof that the system approaches the stability threshold. Therefore, monitoring damping in an on-line identification scheme can give good indication about the dynamical state of the system.

The motion of an  $n$  degree-of-freedom system excited by a random excitation  $f(t)$  can be represented by a system of second-order differential equations

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = f(t) \quad (7)$$

$[y_1(t), y_2(t), \dots, y_j(t), \dots, y_n(t)]$  is the vector of  $n$  displacements of the system,  $y_j(t)$  is the displacement of the mass  $j$ . The problem is to calculate the  $n$  operational frequencies,  $(\omega_{op})_j$  and the  $n$  operational damping ratios,  $(\xi_{op})_j, j=1 \dots n$ .

Let  $y_j(k\Delta T)$ ,  $k=0, 1, 2 \dots$  be the discrete samples of the displacement of the  $j$ -mass.  $\Delta T$  is the sampling interval. Then the observations  $y(k\Delta T)$  can be represented by an ARMA model:

$$\sum_{i=0}^p a_i y(t-i) = \sum_{i=0}^q b_i x(t-i), a_0 = 1 \quad (8)$$

The AR characteristic equation of (8) can be written

$$\sum_{i=0}^p a_i y(t-i) = \prod_{j=1}^n (\mu - \mu_j)(\mu - \mu_j^*) \quad (9)$$

where

$$\begin{aligned} \mu_j &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T + i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) \\ \mu_j^* &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T - i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) \end{aligned} \quad (10)$$

$\mu_j^*$  is the complex conjugate of  $\mu_j$  and  $i = \sqrt{-1}$ .

From Eq. (10) can following expressions be derived

$$\begin{aligned} \ln \mu_j &= -(\omega_{op})_j (\xi_{op})_j \Delta T + i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T \\ \ln \mu_j^* &= -(\omega_{op})_j (\xi_{op})_j \Delta T - i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T \end{aligned} \quad (11)$$

By adding the two equation in (11)

$$\ln \mu_j + \ln \mu_j^* = -2(\omega_{op})_j (\xi_{op})_j \Delta T$$

or

$$\ln(\mu_j \mu_j^*) = -2(\omega_{op})_j (\xi_{op})_j \Delta T \quad (12)$$

By adding the two equation in (10)

$$\begin{aligned} \mu_j + \mu_j^* &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) + \\ &+ \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) = \\ &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \cdot \dots \\ &\dots \left[ \exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) + \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) \right] \end{aligned} \quad (13)$$

By subtracting the two equation in (10)

$$\begin{aligned} \mu_j - \mu_j^* &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) - \\ &- \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) = \\ &= \exp(-(\omega_{op})_j (\xi_{op})_j \Delta T) \cdot \dots \\ &\dots \left[ \exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) - \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) \right] \end{aligned} \quad (14)$$

Finally, divide Eq. (14) to Eq. (13) to obtain

$$\begin{aligned} \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} &= \\ &= \frac{\exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) - \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T)}{\exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) + \exp(-i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T)} \end{aligned}$$

and

$$\begin{aligned} \exp(i(\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) &= \\ &= \cos((\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) + j \sin((\omega_{op})_j \sqrt{1 - (\xi_{op})_j^2} \Delta T) \end{aligned}$$

$$\begin{aligned} & \exp(-i(\omega_{op})_j \sqrt{1-(\xi_{op})_j^2} \Delta T) = \\ & = \cos((\omega_{op})_j \sqrt{1-(\xi_{op})_j^2} \Delta T) - j \sin((\omega_{op})_j \sqrt{1-(\xi_{op})_j^2} \Delta T) \end{aligned}$$

Then

$$(\omega_{op})_j \sqrt{1-(\xi_{op})_j^2} \Delta T = \tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right) \quad (15)$$

Divide now Eq. (12) to Eq. (15)

$$\frac{\ln(\mu_j \mu_j^*)}{\tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right)} = -2 \frac{(\xi_{op})_j}{\sqrt{1-(\xi_{op})_j^2}}$$

or

$$\begin{aligned} \ln(\mu_j \mu_j^*) \sqrt{1-(\xi_{op})_j^2} &= -2(\xi_{op})_j \tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right) \\ \ln(\mu_j \mu_j^*)^2 (1-(\xi_{op})_j^2) &= -4(\xi_{op})_j^2 \left[ \tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right) \right]^2 \\ (\xi_{op})_j &= \frac{\ln(\mu_j \mu_j^*)}{\sqrt{\ln(\mu_j \mu_j^*)^2 - 4 \left[ \tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right) \right]^2}} \quad (16) \end{aligned}$$

Replacing  $(\xi_{op})_j^2$  in (12)  $(\omega_{op})_j$  can be calculated as

$$(\omega_{op})_j = -\frac{1}{2\Delta T} \sqrt{\ln(\mu_j \mu_j^*)^2 - 4 \left[ \tan^{-1} \left( \frac{\mu_j - \mu_j^*}{\mu_j + \mu_j^*} \right) \right]^2} \quad (17)$$

The irrefutable advantages of the approach presented in this section are summarized below:

- Provide a robust tool for discrimination between forced and self-exciting oscillations.
- Tracking of the time-varying dynamics and hereby readily to be implemented in recursive schemes for real time identification and control.
- Capable of “directly” capturing the underlying structural dynamics responsible for the non-stationary behaviour and for further characterizing the process performance in term of quality and productivity.
- Parsimonious management of information acquired as models may be represented by a limited number of parameters.
- Flexibility in fault diagnosis, as they allow for the use of the broad class of parametric diagnosis techniques. Extension to other purposes such as development of diagnostic technique for machine tool maintenance.

### 3. EXPERIMENTS AND RESULTS

A series of experiments were carried out with the purpose of investigating the capability of model-based identification and in particular ARMA models to capture the dynamics of machining system. The model-based identification method is applied for analysis of face milling of prehardened steel. This work material with hardness of 46HRC is used for manufacturing of moulds and dies and therefore its machining requires a careful control of surface characteristics. Both off-line and in-process identification methods were used to identify the physical parameters describing the machining dynamics.

#### 3.1 Experiment setup

The machine tool used in the milling experiment was a vertical three-axes machining center with a 5000 rpm spindle equipped with an ISO 50 taper. The acoustic sound [12] and the three-component vibration from the spindle were recorded during machining operations. The milling cutter (L = 70 mm and D = 20 mm) was a three-tooth solid carbide end mill. Coated inserts (nose radius: 1.6 mm) with grade R390-11T3 16E-PM GC 1030 equipped the cutter. Cutting parameters were feed: 0.12 mm/tooth, spindle speed: 2200, 2300 and 2400 rpm, width of cut: 0.5, 1, 1.5, 2 and 3 mm. The workpiece, Fig. 1, was made of prehardened steel Toolox® 44. To ensure evenly distributed clamping forces, a magnetic table was used as fixture. Prismatic workpieces with the total dimension of 250x200x60 mm were ground on the bottom side to ensure a good contact condition on the magnetic table and were prepared for stepwise increasing of the axial depth of cut from 1 to 8 mm (see Fig. 1).

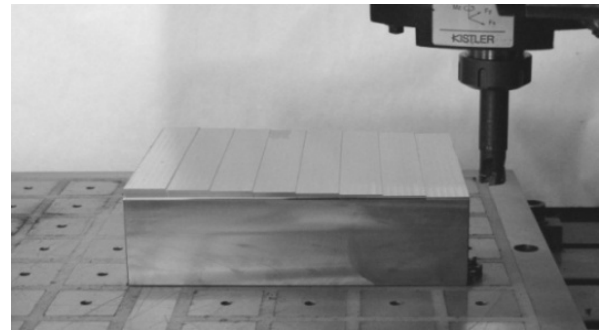


Fig. 1: Workpiece was prepared for stepwise increasing of axial depth of cut between 1 mm and 8 mm during each run.

As known, chatter is always generated close to a structural natural frequency. Instability is likely to occur in the weakest mode or modes of the structure. Normally these modes can be related machine tool structure such as tool, tool holder and spindle.

Experimental Modal Analysis (EMA) was used to identify those modes natural frequency and damping ratio. The first natural frequency mode is about 1400 Hz and is related to the system tool – tool holder and while the second natural frequency mode is about 2200 Hz and is related to the tool and tool clamping. In Fig. 2 the power spectrum (waterfall diagram), representing acquired acoustic signal from down milling test with 1 mm width of cut and spindle speed of 2200 rpm, is illustrated. The figure clearly shows power concentrations around tooth pass frequency (and its harmonics), tool- and tool – tool holder operational frequencies (compare with EMA). The tooth passing frequency and its harmonics, corresponding to the forced excitation, is the dominant frequency during stable machining. By applying a bandpass filter and separating the tooth passing frequency from the tool and tool holder operational frequencies, lower model order can be used to detect operational parameters.

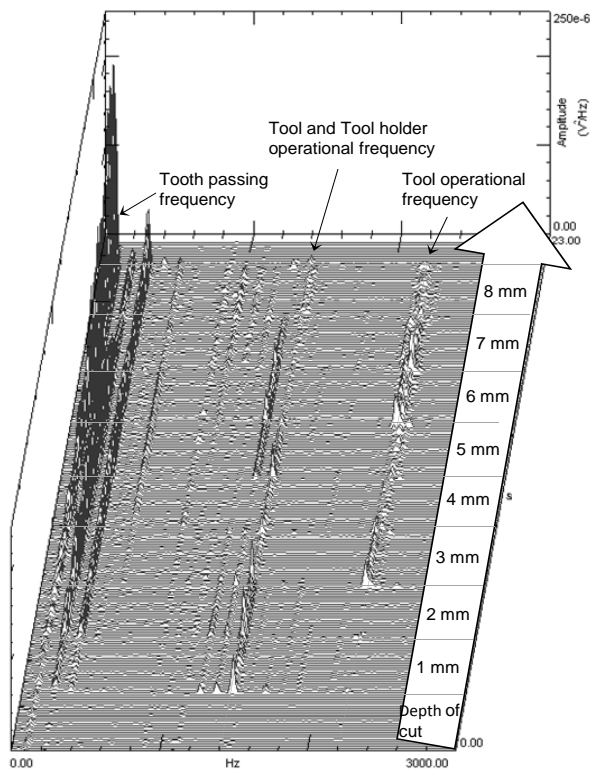


Fig. 2: Power spectrum (waterfall diagram) over machining sound. It shows 23 seconds machining with increasing depth of cut. It increases in steps of 1 mm to a maximum of 8 mm.

**3.2. Off-line ARMA modelling and parameter estimation**

ARMA models of an order determined by the AIC informatics criterion [1] are fitted to the acquired signals by help of Gauss-Newton algorithm [6]. From the estimated model parameters, dynamic characteristics of the machining system are calculated.

In the off-line identification approach the response signal is fitted into eight different models corresponding to each step of depth of cut (1 to 8 mm). Samples are always taken from the middle of each step to ensure stable vibratory levels. The reliability of the identified models is reflected by either plotting the residual curves for each model or by evaluating the standard deviation for each parameter. In Table 1 model parameters along with the standard deviation are displayed for the real machining data. The standard deviation for each individual AR ( $a_1$  to  $a_8$ ) parameter is in largest case ( $a_8$ )  $\pm 0.23\%$ . This reflects a good representation of data.

Table 1. ARMA 8,7 parameters representing real machining

AR parameters representing real machining			
$a_1$	$a_2$	$a_3$	$a_4$
-5.951 $\pm 0.0023$	+17.05 $\pm 0.0121$	-30.2 $\pm 0.0210$	+35.98 $\pm 0.0453$
$a_5$	$a_6$	$a_7$	$a_8$
-29.47 $\pm 0.0447$	+16.23 $\pm 0.0289$	-5.528 $\pm 0.0113$	+0.9062 $\pm 0.0021$
MA parameters representing real machining			
$m_1$	$m_2$	$m_3$	$m_4$
+1.552 $\pm 0.0064$	+1.613 $\pm 0.0080$	+1.242 $\pm 0.0168$	+0.2331 $\pm 0.0143$
$m_5$	$m_6$	$m_7$	
-0.1136 $\pm 0.0169$	-0.1767 $\pm 0.0070$	-0.0104 $\pm 0.0067$	

The dynamic parameters, frequencies ( $f_{opj}$ ) and damping ratio ( $\xi_{opj}$ ) are determined by solving the corresponding ARMA characteristic equation of the recorded acoustic sound. Then each pair of roots corresponds to a mode of vibration defined by the operational- frequency and damping ratio. The most significant three identified pairs of an ARMA 8,7 model are presented in Table 2 and depicted in Fig.3.

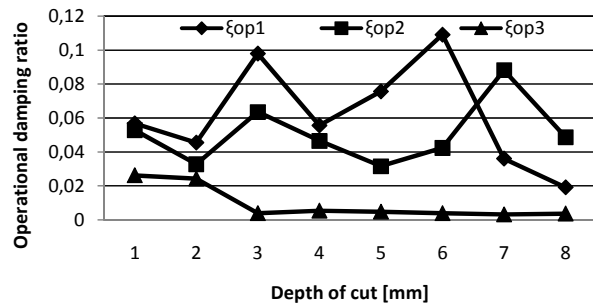


Fig. 3: Identified operational damping ratios from machining with stepwise increasing depth of cut, each model at three frequencies.

### 3.3. In-process parameter estimation based on recursive ARMA modelling

Recursive or sequential estimation of ARMA model parameters is a critical condition for in-process monitoring and control of machining systems. As discussed earlier, estimation of the key dynamic parameter, operational frequencies and damping ratios, offers a robust criterion for discrimination between systems of different types. In an in-process model identification scheme, these dynamic parameters can be used to monitor the stability of a machining system or to optimize stable cutting conditions with respect to surface roughness.

As the sampling frequency can be rather high in milling (12 kHz or higher), the implementation of recursive algorithms requires substantial efforts in terms of computational resources and accuracy, as well as the convergence and stability of the algorithms. In order to track time-varying machining systems, the identification algorithm must be more ‘alert’. Since the parameter estimation is sensitive to random disturbances in the measurements, the design parameters controlling the trade-off between alertness and noise sensitivity must be carefully adjusted.

The dynamic properties of the system’s response are computed from the ARMA model transfer function by separating the AR component [8]. In the down milling experiment with acoustic sound recorded as shown in Fig. 4a, the axial depth of cut is stepwise increased from 1 to 8 mm. Due to the increasing of the depth of cut, the system approaches the stability limit. Three critical frequencies are identified in a recursive ARMA 6,5 identification scheme, ~1100 Hz corresponding to the combined tool-tool holder-spindle natural frequency, ~1400 Hz corresponding to the combined tool-tool holder natural frequency, and ~2200 Hz corresponding to the tool natural frequency. In Fig. 4b the tracking of the variation of the second operational frequency and the corresponding damping with respect to the variation of the depth of cut are illustrated. Figure 4c shows similar variation of the third operational frequency (tool frequency) and the corresponding damping ratio. The dynamics of the machining system revealed in the latter figure is dominated by the tool vibration

mode. The damping variation computed from the ARMA models is able to track in real time the variation of the system dynamics. As the system approaches the limit of stability the damping ratio is close to zero. It is important to stress again that the damping ratio computed from the ARMA models represent the overall damping, i.e. joint system, elastic structure-cutting process.

Finally it must be mentioned that the results of the recursive identification were very similar to those produced by the off-line identification which demonstrates the high reliability of the presented procedure.

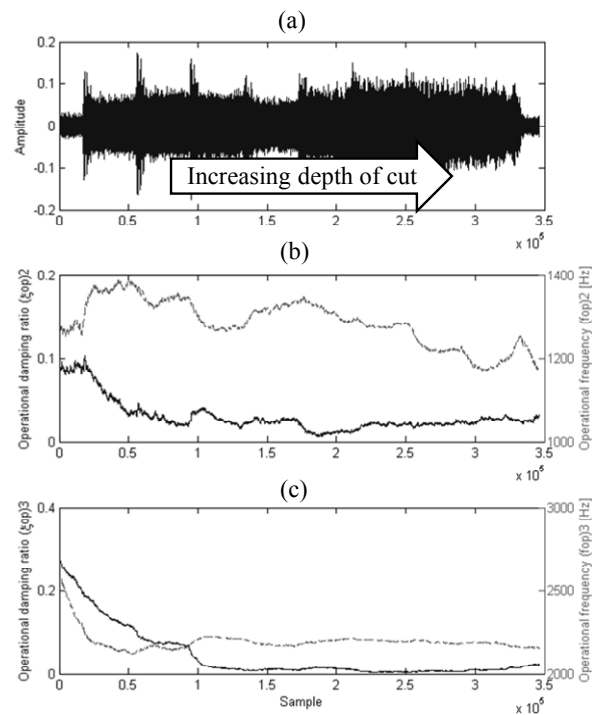


Fig. 4: In-process identification of a face mill operation with depth of cut variation from 1 to 8 mm. (a) Sound pressure level measured by a microphone, (b) tracking the variation of the second operational frequency and damping ratio and (c) tracking the variation of the third operational frequency and damping ratio.

Table 2. Off-line identified dynamic parameters, frequencies ( $f_{op}j$ ) and damping ratios ( $\zeta_{op}j$ )

Depth of cut $a$ [mm]	Operational frequency ( $f_{op}1$ ) [kHz]	Operational damping ratio ( $\zeta_{op}1$ )	Operational frequency ( $f_{op}2$ ) [kHz]	Operational damping ratio ( $\zeta_{op}2$ )	Operational frequency ( $f_{op}3$ ) [kHz]	Operational damping ratio ( $\zeta_{op}3$ )
1	1.11	0.057	1.46	0.053	2.16	0.026
2	1.08	0.046	1.42	0.033	2.20	0.024
3	1.09	0.098	1.41	0.063	2.21	0.0039
4	1.02	0.056	1.46	0.045	2.21	0.0054
5	1.04	0.077	1.40	0.032	2.23	0.0048
6	0.97	0.110	1.34	0.042	2.21	0.0039
7	1.07	0.036	1.50	0.088	2.22	0.0032
8	1.03	0.019	1.47	0.048	2.22	0.0037

**3.4. ARMA parameter estimation for surface roughness characterization**

The produced roughness is the sum of two independent effects: (1) the ideal surface roughness as a result of the geometry of tool and feed rate and (2) the natural surface roughness as a result of the irregularities in the cutting operation. Vibration of the machine tool, defects in the work material, wear of the tool or variability in chip formation contribute to the irregularities of the surface. In a stable milling operation the vibration period for each cutting edge is equal to the tooth passing period. In such conditions, the configuration of material removal is the same for each tooth, giving an equal cutting force. If, however, the milling operation is unstable, the surface profile will be ‘modulated’ by frequencies close to the natural frequencies of the weakest mode or modes in the combined system tool-tool holder-spindle-workpiece (normally represented by the tool or tool holder modes). In the surface profiles for stable and unstable (see Fig. 5) down milling this occurrence can be noticed.

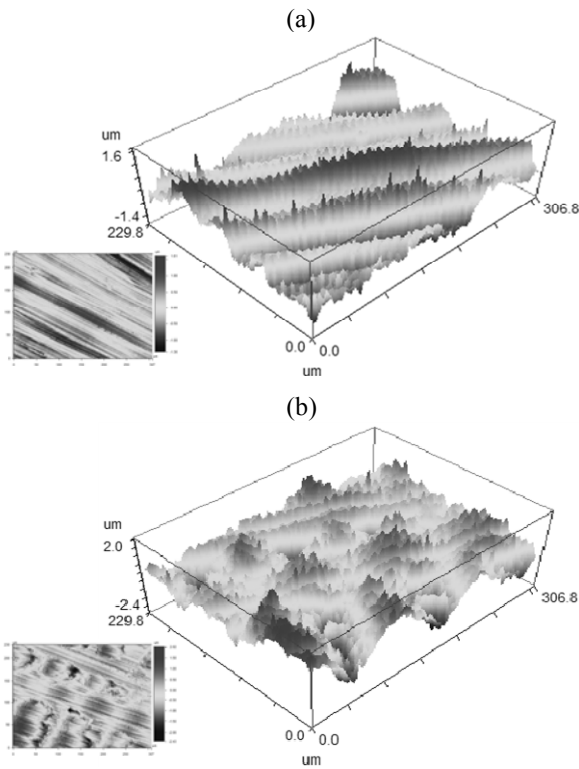


Fig. 5: (a) Surface profile for stable and (b) unstable face milling operation.

In the following experiment, the identified dynamic parameters of a down milling operation are correlated to surface roughness. Width of cut was chosen to 1.5 mm and feed per tooth was 0.12 mm. As the machining proceeds, operational damping ratio variations follow those of the depth of cut. As shown

in Table 3 and visualised in Fig. 6, the damping ratio accurately tracks the variations in the surface finish.

Table 3. Correlation between operational damping ratio and surface roughness

Depth of cut a [mm]	Operational damping ratio ( $\xi_{op}$ ) <sub>i</sub>	R <sub>a</sub> [ $\mu$ m]	R <sub>z</sub> [ $\mu$ m]
1	0.097	0.22	1.47
2	0.064	0.21	1.57
3	0.036	0.81	3.74
4	0.035	1.23	4.39
5	0.060	0.29	1.86
6	0.087	0.23	1.31
7	0.080	0.20	1.26
8	0.072	0.23	1.50

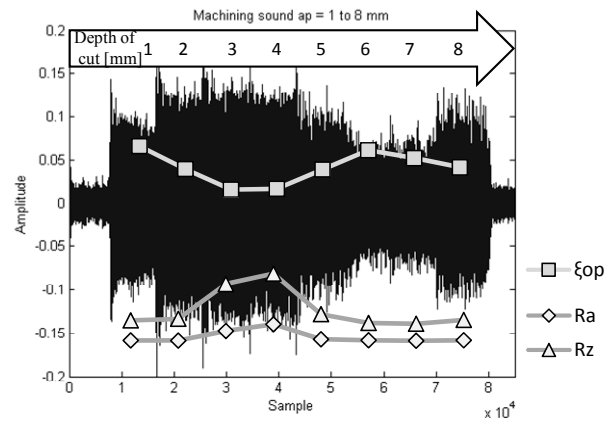


Fig. 6: Calculated operational damping ratio ( $\xi_{op}$ )<sub>i</sub> and the surface roughness Ra, Rz of the machined workpiece visualised on the acoustic sound produced during machining.

**3.5. Classification of machining states based on ARMA parameters**

In this section we study how AR parameters from ARMA models can be used to discriminate between different machining system states. The concept used here for classification is by distance functions. Pattern classification by distance functions is a concept well developed in automatic pattern recognition. Considering M pattern classes and assuming that these classes are represented by prototype pattern  $z_1, z_2, \dots, z_m$ , the Euclidean distance between an arbitrary pattern vector  $x$  and the  $i$ th prototype is given by equation

$$D_i = \|x - z_i\| = \sqrt{(x - z_i)'(x - z_i)} \quad (18)$$

A minimum-distance classifier computes the distance from a pattern  $x$  of unknown classification to the prototype of each class, and assigns the pattern to the class to which is closest. In the case of three-class, spindle idle (no machining), machining at 1 mm depth

of cut and machining at 2 mm depth of cut, the linear decision surface separating every pair of prototype points  $z_i$  and  $z_j$  is the hyperplane which is perpendicular on the line segment joining the two points. In the case presented in this paper the decision plane is normal to the line connecting the centroid points calculated as the mean values of the all points belonging to a certain class.

Figure 7 shows the classification results from modeling of several machining test. Machining and spindle idle states show clear clustering properties. The three patterns, spindle idle (off machining), machining at 1 mm depth of cut and machining at 2 mm depth of cut are perfectly disjoint. In all tests the workpiece dimensions and materials were similar.

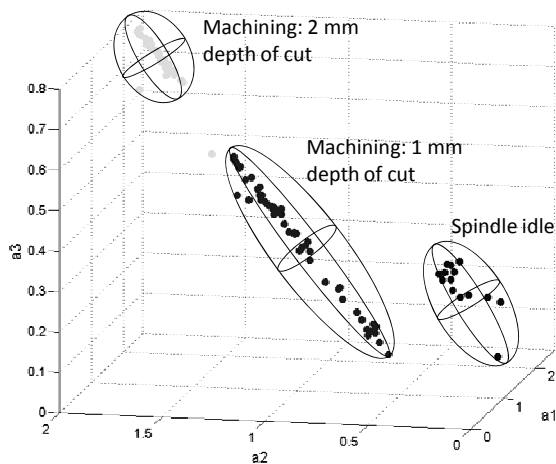


Fig. 7: Classification results from modelling several machining systems states. Identified model parameters ( $a_1$ ,  $a_2$  and  $a_3$ ) representing a machining system going from spindle idle to machining state.

#### 4. DISCUSSION

Compared with other methods for analysis of machining systems, the implementation of model based identification brings the following benefits:

- Able to capture the dynamic interaction between the elastic structure and machining process, therefore can be readily applied in normal cutting operations.
- Operational dynamic parameters, frequency and damping, can be straightforward computed from model's parameters and used as a discrimination criterion between various machining states.
- In the recursive form the method can be used to track time-varying machining systems and hereby used for real time monitoring, control and optimization.
- Capable of 'directly' capturing the underlying structural dynamics responsible for the non-stationary behaviour and for further

characterizing the process performance in term of quality and productivity.

#### 5. CONCLUSIONS

The desired mathematical model of the machining system presented in this paper is based on the data obtained during normal operational conditions. In this way we take a step beyond the classical method of analyzing the dynamics of a machining system, which separately identifies the structural and process parameters.

The model-based identification technique represents a robust approach for the characterization of the dynamic behaviour of machining systems. The results from experiments demonstrate the feasibility of the operational damping ratio (and related frequency) as a measure for tracking the variability of a machining system and to be used as criterion for discrimination between various dynamic systems. The model-based identification provides a robust tool for discrimination between forced and self-exciting oscillations.

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#### REFERENCES

- [1] Akaike, H., *Modern development of statistical methods, Trends and progress in system identification* [Book], 1981, Pergamon Press;
- [2] Archenti, A., *Model-based investigation of machining systems characteristics: static and dynamic stability analysis*, Licentiate thesis, The Royal Institute of Technology, KTH Production Engineering, 2008, Stockholm, ISBN 978-91-7415-196-1;
- [3] Archenti, A., Nicolescu C.M., *Model-based Identification of Dynamic Stability of Machining Systems*, 1st International Conference on Process Machine Interaction - Proceedings, Hannover, Germany, 2008, pag 41-52;
- [4] Bejhem M., *Machining monitoring and control*, PhD thesis, The Royal Institute of Technology, KTH Production Engineering, 2001, Stockholm, ISSN 1650-1888;
- [5] Brockwell, P.J., Davis, R.A., *Time series: theory and methods* [Book], 1987, Springer-Verlag;
- [6] Ljung, L., *System Identification: Theory for the User* [Book], 2006, - Englewood, New Jersey : Prentice-Hall Inc;
- [7] Nicolescu, C.M., *Analysis, Identification and Prediction of Chatter in Turning*, PhD thesis, The Royal Institute of Technology, KTH Production Engineering, 1991, Stockholm, ISSN 0284 0537;
- [8] Pandit, S.M., Wu, S.M., *Time series and system analysis with application* [Book], 1983, Wiley;
- [9] Smith, S., Tlusty, J., *Efficient simulation programs for chatter in milling*, General Assembly of CIRP, Vol. 43, 1993, Edinburgh;
- [10] Tarantola, A., *Inverse Problem Theory and Model Parameters Estimation*, 2005, SIAM, Philadelphia;
- [11] Tlusty, J., Zaton, W., Ismail, F., *Stability lobes in milling*, CIRP Annals, v 32, 1983, n 1, pag 309-313;
- [12] Tlusty, J., Andrews, G.C., *Critical review of sensors for unmanned machining*, CIRP Annals, v 32, n 2, 1983, pag 563-572;
- [13] Tlusty, J., Smith, S., Zamudio, C., *Evaluation of cutting performance of machining centers*, CIRP Annals, v 40, n 1, 1991, pag 405-410;