BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică "Gh. Asachi" din Iaşi
Tomul LIV (LVIII), Fasc. 1, 2008
Secţia
CONSTRUCŢII DE MAŞINI

RACK-GEAR TOOL PROFILING BY BEZIER POLYNOMIALS

II. INVOLUTE AND TROCHOIDAL PROFILE

BY

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Abstract. In this paper are presented application of the approximation by Bezier polynomials for the rack-tool's profiling at generation of profiles with variable curvature, as involute and trochoidal. Are presented examples regarding the error approximation level regarding profiles determined based on the fundamentals laws of surfaces enwrapping.

Key words: surface enveloping, approximate profiling, Bezier polynomials, rack-gear profiling.

2000 Mathematics Subject Classification: 53B25, 53C15

1. Introduction

It was presented algorithms for rack-gear tool's profiling by Bezier polynomials for generation of straight line and circle arc profiles, belongs to an ordered profiles curl.

These elementary profiles belong to composed profiles, which, are usually generated by rolling.

They are frequent used profiles with variable curvature: involute profile (the involute of a circle with R_b radius) and trochoidal profiles.

In paper is proposed an extension of the profiling method for the rack-gear tool and, from here, of the worm cutter tool for these profiles: involutes and trochoidal.

2. Involute arc — Algorithm

In the relative reference systems (see figure 1), are defined the parametrical equations of the R_b radius circle involute:

(1) $$E\left|\begin{array}{l} X(q) = -R_b \cdot \cos q - R_b \cdot q \cdot \sin q; \\ Y(q) = R_b \cdot \sin q - R_b \cdot q \cdot \cos q. \end{array}\right.$$

The variation limits of the q parameter are established regarding the internal (R_i) and external (R_e) radii, between are extended the profile:

(2) $$q_A = \frac{\sqrt{R_i^{\,2} - R_b^{\,2}}}{R_b} \text{ and } q_B = \frac{\sqrt{R_e^{\,2} - R_b^{\,2}}}{R_b}.$$

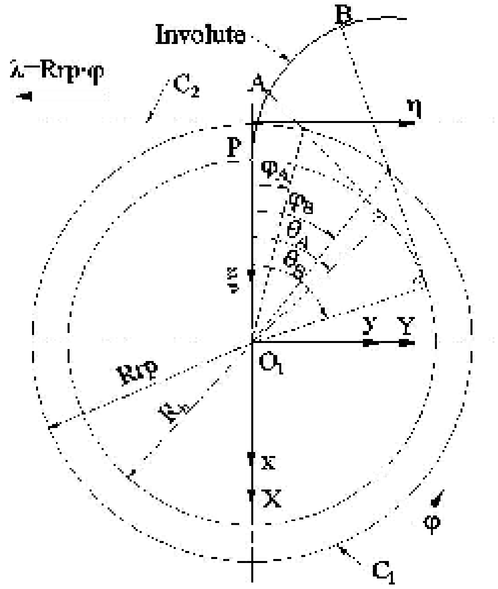

Fig. 1. – Involute arc profile associated with the rolling centrodes couple

From the condition that the involute's normal,

(3) $$\vec{N}_E : [X - X(q)](-\cos q) + [Y - Y(q)]\sin q = 0,$$

must intersect with the rolling circle:

(4) $$C_1\left|\begin{array}{l} X = -Rrp \cdot \cos j; \\ Y = Rrp \cdot \sin j, \end{array}\right.$$

with j variable parameter, result the equation

(5) $$j = \arccos\left[\frac{R_b}{R_{rp}}\right] + q,$$

representing the enwrapping condition. For a substitution polynomial of 3rd degree, are defined the values for the substitution curve poles coordinates, see table 1.

Also, in the table 1, are presented the calculus relations for the 3rd degree Bezier polynomials coefficients, specifically for the considered elementary profile type.

Table 1.
Involute arc, 3rd degree approximation polynomial coefficients identification

θ	**Primary profile**	**Enwrapping condition**
θ_A	$X_A = -R_b \cdot \cos q_A - R_b \cdot q_A \cdot \sin q_A$ $Y_A = R_b \cdot \sin q_A - R_b \cdot q_A \cdot \cos q_A$	$j_A = \arccos\left[\frac{R_b}{Rrp}\right] + q_A$
θ_C	$q_C = q_A + \frac{q_B - q_A}{3}$ $X_C = -R_b \cdot \cos q_C - R_b \cdot q_C \cdot \sin q_C$ $Y_C = R_b \cdot \sin q_C - R_b \cdot q_C \cdot \cos q_C$	$j_C = \arccos\left[\frac{R_b}{Rrp}\right] + q_C$
θ_D	$q_D = q_A + 2\frac{q_B - q_A}{3}$ $X_D = -R_b \cdot \cos q_D - R_b \cdot q_D \cdot \sin q_D$ $Y_D = R_b \cdot \sin q_D - R_b \cdot q_D \cdot \cos q_D$	$j_D = \arccos\left[\frac{R_b}{Rrp}\right] + q_D$
θ_B	$X_B = X_{O_C} + r \cdot \cos q_B$ $Y_B = Y_{O_C} - r \cdot \sin q_B$	$j_B = \arccos\left[\frac{R_b}{Rrp}\right] + q_B$
λ	**Points on the rack-gear profile**	**Approximation polynomial coefficients**
0	$x_A = X_A \cos j_A - Y_A \sin j_A + R_{rp}$ $h_A = X_A \sin j_A + Y_A \cos j_A + R_{rp} \cdot j_A$	$D_x = x_A$ $D_h = h_A$
1/3	$x_C = X_C \cos j_C - Y_C \sin j_C + R_{rp}$ $h_C = X_C \sin j_C + Y_C \cos j_C + R_{rp} \cdot j_C$	$C_x = \frac{18 \cdot x_C - 9 \cdot x_D + 2 \cdot x_B - 5 \cdot x_A}{6}$ $C_h = \frac{18 \cdot h_C - 9 \cdot h_D + 2 \cdot h_B - 5 \cdot h_A}{6}$
2/3	$x_D = X_D \cos j_D - Y_D \sin j_D + R_{rp}$ $h_D = X_D \sin j_D + Y_D \cos j_D + R_{rp} \cdot j_D$	$B_x = \frac{-5 \cdot x_B + 2 \cdot x_A + 18 \cdot x_D - 9 \cdot x_C}{6}$ $B_h = \frac{-5 \cdot h_B + 2 \cdot h_A + 18 \cdot h_D - 9 \cdot h_C}{6}$
1	$x_B = X_B \cos j_B - Y_B \sin j_B + R_{rp}$ $h_B = X_B \sin j_B + Y_B \cos j_B + R_{rp} \cdot j_B$	$A_x = x_B$ $A_h = h_B$

3. Trochoidal arc — Algorithm

The trochoid described by a point M, from the r rolling center, which rolling on the circle with radius R has equations:

(6) $$\Sigma\left|\begin{array}{l} X = r\cdot\cos(q+y) - (R+r)\cos y\ ; \\ Y = -r\cdot\sin(q+y) + (R+r)\sin y\ , \end{array}\right.$$

with

(7) $$q = \frac{R}{r}y$$

where y is the variable angular parameter, measures on the circle with radius *R*.

From intersection condition between the normal at Σ trochoidal curve and the $Rrp \equiv R$ circle,

(8) $$\overrightarrow{N_\Sigma} : [X - X(y)]X'_y + [Y - Y(y)]Y'_y = 0\,,$$

(9) $$C_1\left|\begin{array}{l} X = Rrp\cdot\cos j\ ; \\ Y = Rrp\cdot\sin j\ , \end{array}\right.$$

with j—parameter in the rolling movement, result the condition

(10) $$j = y\ .$$

For A and B points on the trochoidal curve, results the enveloping approximation polynomial elements, see table 2 and figure 2.

The profile substitution Bezier polynomial coefficients are determined as in the previous case, see table 1.

The calculus relations of the 3[rd] degree approximation polynomial are the same as in table 1.

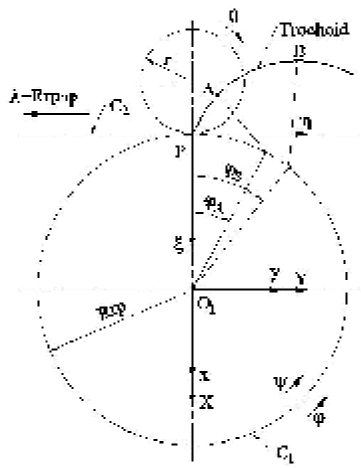

Fig. 2. – Trochoidal arc profile associated with the rolling centrodes couple

Table 2.
Trochoidal curve arc, 3rd degree approximation polynomial coefficients identification

θ	**Primary profile**	**Enwrapping condition**
θ_A	$X_A = r\cdot\cos(q_A + y_A) - (R+r)\cdot\cos y_A$ $Y_A = -r\cdot\sin(q_A + y_A) + (R+r)\cdot\sin y_A$ $q_A = \frac{R}{r} y_A$	$j_A = y_A$
θ_C	$X_C = r\cdot\cos(q_C + y_C) - (R+r)\cdot\cos y_C$ $Y_C = -r\cdot\sin(q_C + y_C) + (R+r)\cdot\sin y_C$ $q_C = \frac{R}{r} y_C$ $y_C = y_A + \frac{\lvert y_B\rvert - \lvert y_A\rvert}{3}$	$j_C = y_C$
θ_D	$X_D = r\cdot\cos(q_D + y_D) - (R+r)\cdot\cos y_D$ $Y_D = -r\cdot\sin(q_D + y_D) + (R+r)\cdot\sin y_D$ $q_D = \frac{R}{r} y_D$ $y_D = y_A + 2\frac{\lvert y_B\rvert - \lvert y_A\rvert}{3}$	$j_D = y_D$
θ_B	$X_B = r\cdot\cos(q_B + y_B) - (R+r)\cdot\cos y_B$ $Y_B = -r\cdot\sin(q_B + y_B) + (R+r)\cdot\sin y_B$ $q_B = \frac{R}{r} y_B$	$j_B = y_B$

4. Numerical examples

In figure 3 and table 3, are presented the form and coordinates for rack-gear tool's reciprocally enveloping with an involute profile with $A[-140,9;0]$; $R_i = 142$ mm; $R_e = 160$ mm; $R_b = 150$ mm; $R = 150$ mm. The maximum error is $e = 0.0208$ mm.

Table 3

l	Approximated tool profile		Theoretical tool profile		Error	*j*
	x [mm]	*h* [mm]	*x* [mm]	*h* [mm]	[mm]	[rad]
0	**11.6295**	**-2.7423**	**11.6295**	**-2.7423**	**0**	**-0.347**
0.05	7.3435	-1.1825	7.3255	-1.1759	0.0192	-0.2128
0.3	1.0199	1.119	1.0237	1.1176	0.004	-0.017
0.333	**0.4518**	**1.3258**	**0.4518**	**1.3258**	**0**	**0.0009**
0.35	0.1625	1.4311	0.1683	1.4289	0.0062	0.0097
0.65	-4.0114	2.9501	-4.0125	2.9505	0.0012	0.1399
0.666	**-4.2058**	**3.0209**	**-4.2058**	**3.0209**	**0**	**0.1458**
0.7	-4.6025	3.1653	-4.6071	3.167	0.005	0.1582
0.95	-7.3109	4.151	-7.3113	4.1511	0.0004	0.2412
1	**-7.767**	**4.317**	**-7.767**	**4.317**	**0**	**0.2583**

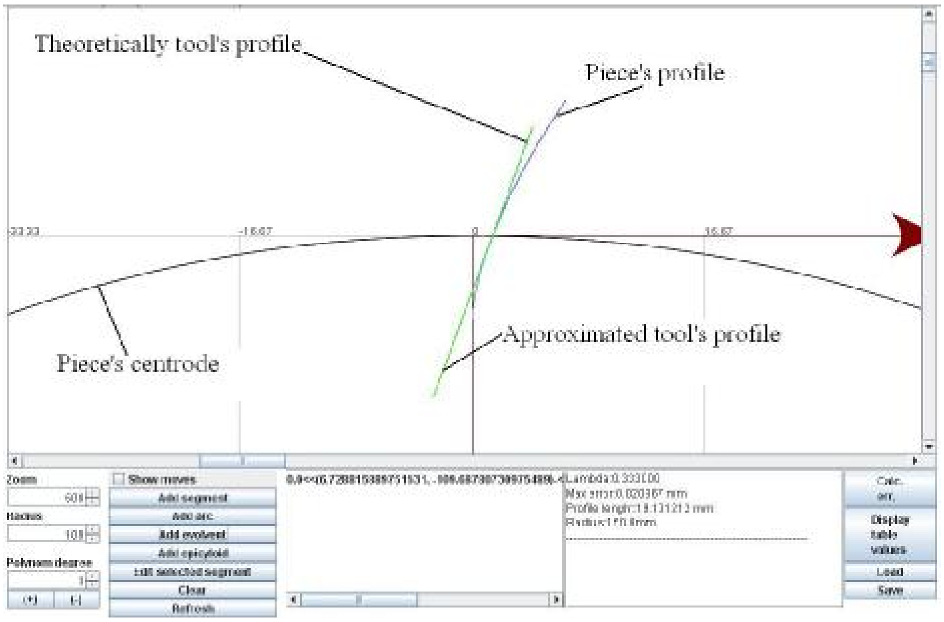

Fig. 3 – Rack-gear approximation for involute profile

In figure 4 and table 4, are presented the form and coordinates for rack-gear tool's reciprocally enveloping with an trochoidal curve profile with $A[-50;0]$; $r = 10$ mm; $R = 50$ mm. The maximum error is $e = 0.023$ mm.

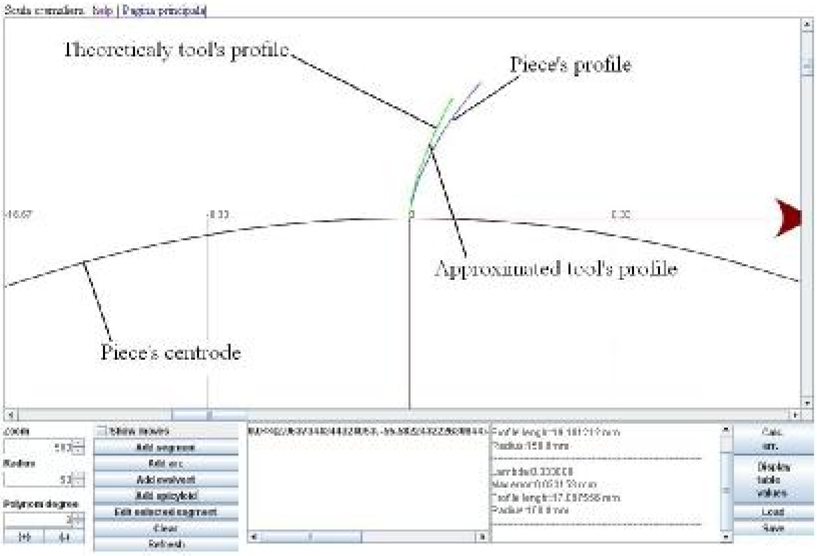

Fig. 4 – Rack-gear approximation for trochoidal profile

Table 4

l	**Approximated tool profile**		**Theoretical tool profile**		**Error** [mm]	*j* [rad]
	x [mm]	*h* [mm]	*x* [mm]	*h* [mm]		
0	**0**	**0**	**0**	**0**	**0**	**0**
0.05	-0.266	0.0297	-0.2693	0.0239	0.0067	0.0449
0.333	**-1.7443**	**0.3613**	**-1.7443**	**0.3613**	**0**	**0.1189**
0.35	-1.8068	0.3807	-1.8059	0.3803	0.001	0.1263
0.666	**-3.4268**	**1.0113**	**-3.4268**	**1.0113**	**0**	**0.1663**
0.7	-3.547	1.067	-3.5464	1.0667	0.0006	0.1767
1	**-4.9216**	**1.7799**	**-4.9216**	**1.7799**	**0**	**0.2165**

5. Conclusions

1. The presented method, although approximately, assure a good representation of the rack-gear tool’s profile reciprocally enveloping with an involute and trochoidal profiles, which may be part of a composed profiles to be generated.

2. Software dedicated to this algorithm is an instrument which helps to apply this method.

*Acknowledgement***:** This paper was developed under the research programs IDEI, grants ID_656, funded by the Romanian Ministry of Education and Research

Submited: 20th April 2008

Accepted: 30th April 2008

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PROFILAREA PRIN POLINOAME BEZIER A SCULEI-CREMALIERĂ

PROFILURI EVOLVENTICE ŞI CICLOIDALE

(Rezumat)

În lucrare, se prezintă aplicații ale metodei de aproximare prin polinoame Bezier a profilurilor sculei-cremalieră generatoare înfăşurătoare a unor profiluri cu rază de curbură variabilă: evolventa cercului și epicicloida. Se prezintă exemple numerice privind mărimea erorii de aproximare în raport cu profilurile cremalierei determinate în baza legilor fundamentale ale înfăşurării suprafețelor, calculate cu ajutorul unui produs soft original dedicat acestei aplicații.