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PROFILES REPRESENTED BY POLES APPROXIMATION PRECISION IMPROVEMENT WHEN GENERATING WHIRLS OF SURFACES WITH A RACK-TOOL BY ROLLING

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Abstract: *Representation by poles, as a way to substitute analytical or non-analytical profiles, was already used to profile tools for generating enwrapped surfaces by rolling method. In this paper, there are suggested algorithms to achieve an improvement of precision concerning the representation by poles of profiles to be generated, together to tools profiles. Examples are presented in the case of generating several profiles with a rack-tool.*

Key words: *representation by poles, enwrapped surfaces, rack-tool.*

1. INTRODUCTION

The possibility to represent profiles by poles is already known [2]. They have been also presented modalities of using this way of profiles representation to study enwrapping processes concerning whirls of surfaces (profiles) associated to a couple of rolling centrods [5, 6]. Thus, couples of centrods circle - straight line, circle - circle, straight line – circle were studied; they correspond to generating technologies which use rack-type tools, pinion cutters and rotating cutters [3].

The study of enwrapping processes specific to this type of tools was realized based on plain generating trajectories method [4] by establishing, in the end, a simpler scheme to represent both enwrapped profile and enwrapping curve (tool profile) into a polar expression [5]. The specific problems were solved in the cases of elementary profiles: straight-line segment and arc of circle, [6].

The suggested methodology [5] and the examples presented based on it [6] relieved the possibility of using the representation by poles in the case of reciprocal enwrapping pairs of curves, associated to a couple of rolling centrods by highlighting the facility of applying this method in the conditions when particular elements of approximation curves can be used.

However, specifications about the number of poles necessary to realize a correct approximation of enwrapped profiles and the influence of this number onto tools profiles precision of representation (compared to the results found by using classic methods) are missing.

We further analyze, in the cases of established generating proceedings, by using a rack-tool, a pinion cutter or a rotating cutter to generate helical surfaces, based on each method specific kinematics, the modality to profile the tool.

2. RACK-TOOL PROFILING

Considering the case of a plain profile, owning to an ordered whirl of profiles and associated to a circular centrod, C_1, having R_{rp} radius, in rolling motion with the centrod associated to rack-tool, C_2, the following reference systems are defined (Fig. 1):

- *xyz*, as a global system, having Oz as rotation axis;
- *XYZ* – relative system, joint to the ordered whirl of surfaces;
- ξηζ – relative system, joint to the rack-tool.

The following relative motions can be now defined:

$$\xi = \omega_3^T(\varphi)\cdot X - a \tag{1}$$

and its reverse

$$X = \omega_3(\varphi)[\xi + a], \tag{2}$$

where

$$a = \begin{Vmatrix} -R_{rp} \\ -R_{rp}\cdot\varphi \\ 0 \end{Vmatrix} \tag{3}$$

and *X* means the matrix formed from the co-ordinates of Σ curve (from whirl to be generated) current point with

$$X = \begin{Vmatrix} P_X(\lambda) \\ P_Y(\lambda) \end{Vmatrix}. \tag{4}$$

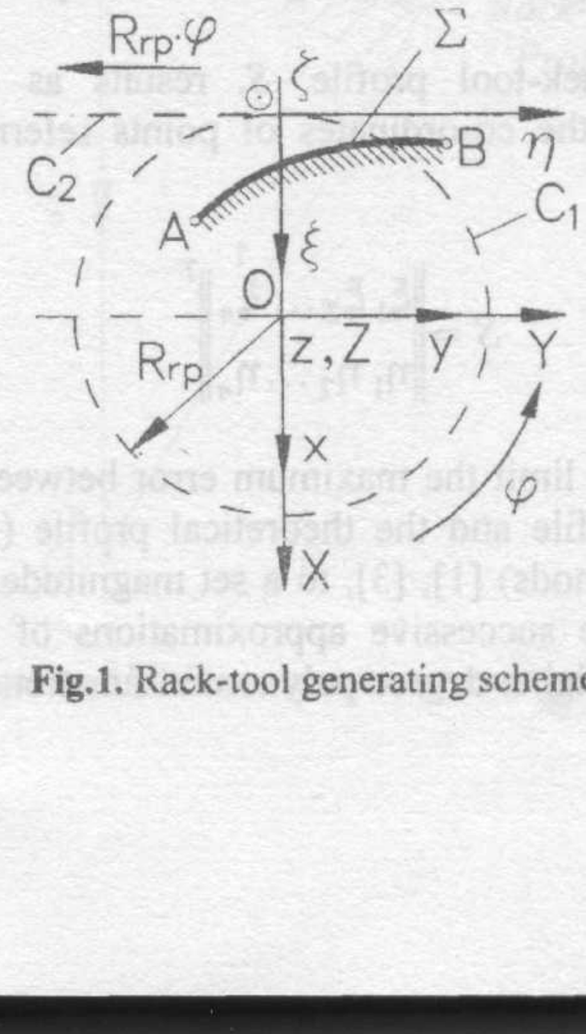

Fig.1. Rack-tool generating scheme.

$P_X(\lambda)$ and $P_Y(\lambda)$ meaning projections on the axis of Σ curve polar representation and λ – variable parameter.

Thus, starting from (1), the family of curves Σ can be determined, into rack-tool reference system:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = P_X(\lambda)\cos\varphi - P_Y(\lambda)\sin\varphi + R_{rp}; \\ \eta = P_X(\lambda)\sin\varphi + P_Y(\lambda)\cos\varphi + R_{rp}\cdot\varphi, \end{vmatrix} \quad (5)$$

where φ is the angular parameter of C_1 centrod rotation motion.

In principle, family of profiles enwrapping curve, expressed in polar manner, represents rack-tool profile, if the enveloping condition is associated to these equations; Gohman expression [1] of the enveloping condition is

$$\vec{N}_\Sigma \cdot \vec{R}_\varphi = 0. \quad (6)$$

In relation (6), the vector $\vec{N}_\Sigma$ means the normal at Σ polar approximated surface

$$\vec{N}_\Sigma = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_{X_\lambda}' & P_{Y_\lambda}' & 0 \\ 0 & 0 & 1 \end{vmatrix} = P_{X_\lambda}'\vec{i} - P_{Y_\lambda}'\vec{j}. \quad (7)$$

Starting from (2), $\vec{R}_\varphi$ vector can be defined, as having the same direction as the speed in the relative motion between the space associated to C_2 centrod and XYZ space,

$$R_\varphi = \frac{dX}{d\varphi} = \dot{\omega}_3(\varphi)\cdot\omega_3^T(\varphi)\begin{Vmatrix} P_X(\lambda) \\ P_Y(\lambda) \end{Vmatrix} + \omega_3(\varphi)\begin{Vmatrix} 0 \\ -R_{rp} \end{Vmatrix}, \quad (8)$$

or, after developing calculus,

$$R_\varphi = \begin{Vmatrix} P_Y(\lambda) + R_{rp}\sin\varphi \\ -P_X(\lambda) - R_{rp}\cos\varphi \end{Vmatrix}. \quad (9)$$

Thus, from (6), (7) and (9), the specific enveloping condition results as

$$[P_Y(\lambda) - R_{rp}\sin\varphi]P_{Y\lambda}' + [P_X(\lambda) + R_{rp}\cos\varphi]P_{X\lambda}' = 0. \quad (10)$$

The ensemble of equations (5) and (10) are giving the rack-tool profile, reciprocal enwrapped to Σ profile, from the whirl of profiles to be generated, represented by poles.

Finally, rack-tool profile, S, results as a matrix formed from the co-ordinates of points referred to $\xi\eta$ system,

$$S = \begin{Vmatrix} \xi_1\ \xi_2 \ldots \xi_n \\ \eta_1\ \eta_2 \ldots \eta_n \end{Vmatrix}^T. \quad (11)$$

In order to limit the maximum error between the approximate profile and the theoretical profile (found by analytical methods) [1], [3], to a set magnitude, we suggest to realize successive approximations of rack-tool profile, S, by higher degree polynomial functions.

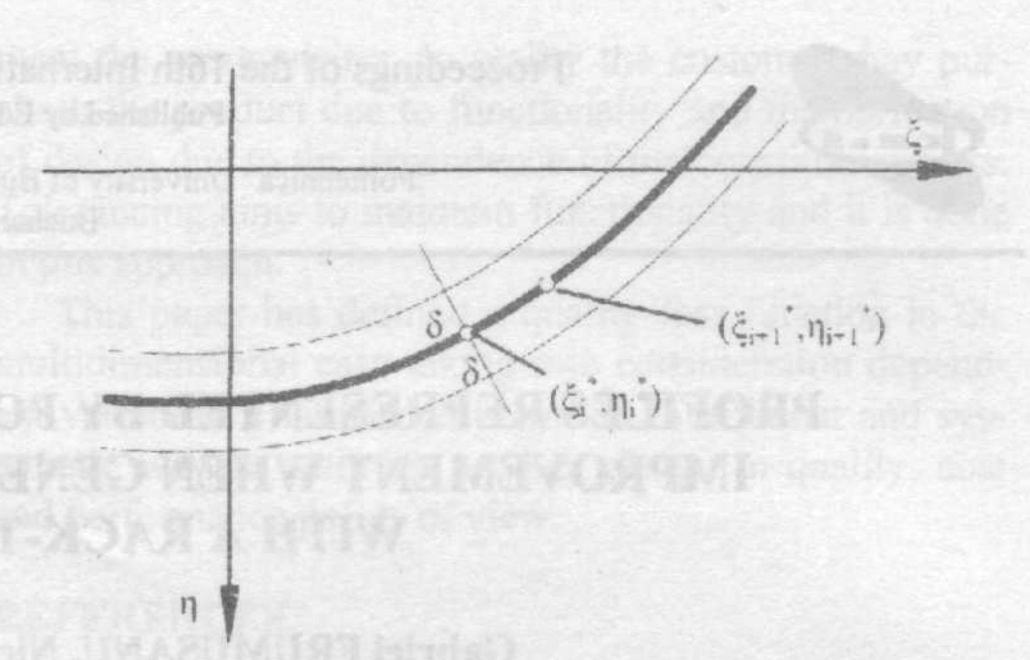

Fig. 2. The tolerance field.

Thus, if tool theoretical profile, S^*, found by one from the fundamental theorems, is represented through the matrix

$$S^* = \begin{Vmatrix} \xi_i^* \\ \eta_i^* \end{Vmatrix}^T \quad (i = 1, 2, \ldots n), \quad (12)$$

then a tolerance field can be established, referred to this profile (Fig. 2), where the approximated profile should found itself.

If we define

$$\tan\beta_i = \frac{\xi_{i+1}^* - \xi_i^*}{\eta_{i+1}^* - \eta_i^*}, \quad (13)$$

then a point $(\xi_i\ \eta_i)$ of approximated profile is in the interior of the tolerance field if

$$\begin{vmatrix} \xi_i \in [\xi_i^* - \delta\cos\beta_i, \xi_i^* + \delta\cos\beta_i] \\ \eta_i \in [\eta_i^* - \delta\sin\beta_i, \eta_i^* + \delta\sin\beta_i] \end{vmatrix} \quad (14)$$

where δ means the maximum admissible error, measured along the normal to the theoretical profile.

Obviously, the deviation from the theoretical profile can be or can be not symmetrical along rack-tool theoretical profile.

The enwrapping curve of Σ profile, given into a polar expression, can be found as S from relation (11), which is approximated by a curve expressed by poles, like it is shown in the scheme from Fig. 3.

By successively making the approximation through polynomial functions of second, third and fourth degree, we can directly observe the effect of increasing polynomial function degree onto the quality of the approximation.

3. NUMERICAL APPLICATIONS

3.1. Rack-bar tool to generate exterior slots

The case of a shaft having exterior 8×52×60 mm slots was considered (Fig. 4). To observe the effect of approximation polynomial function degree increasing, second, third and fourth degree functions were successively used to approximate the same rack-tool theoretical profile. The results are shown through points co-ordinates, respective, in Tables 1, 2 and 3, near the theoretical profile points co-ordinates, to realize a better comparison.

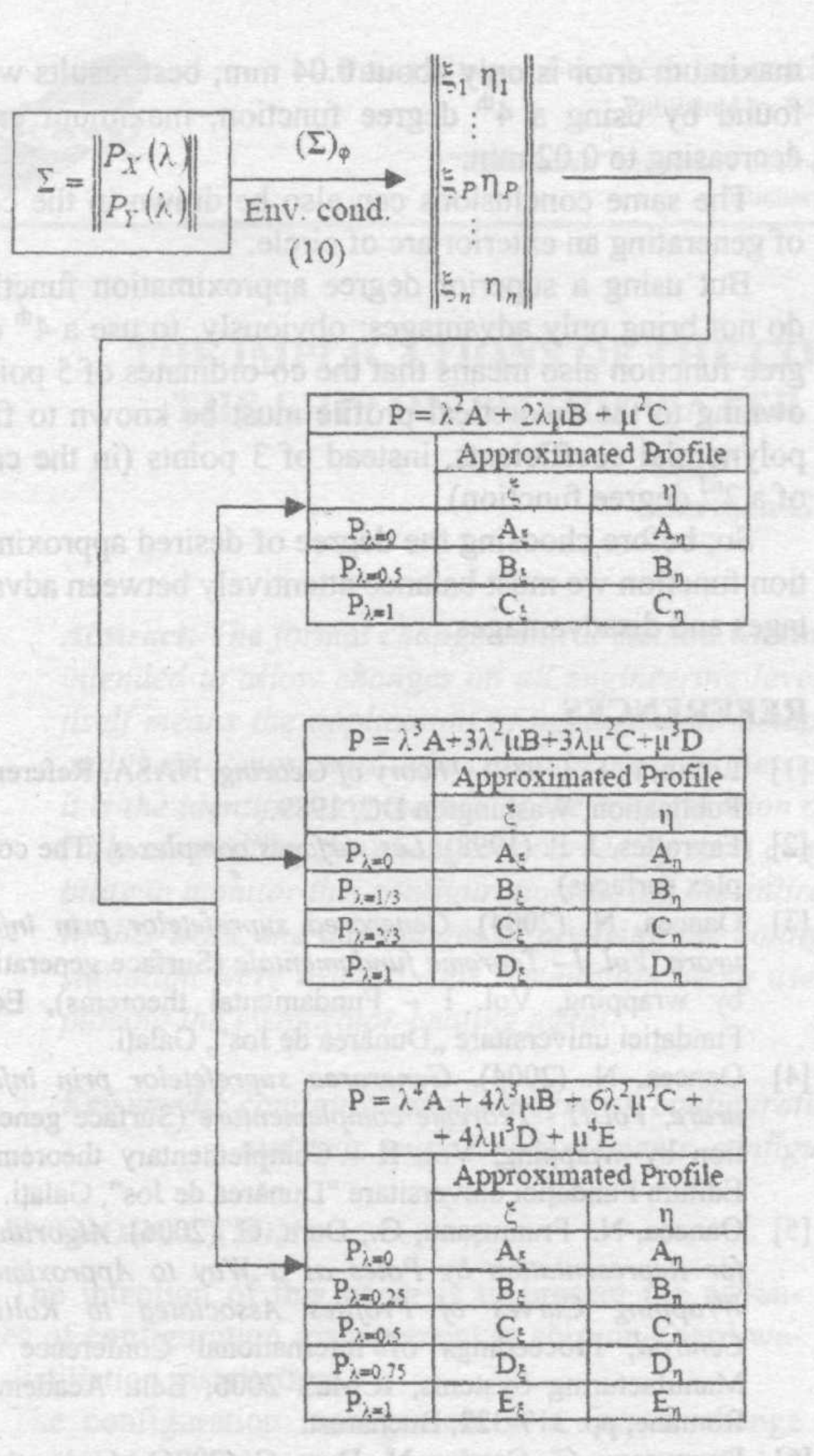

Fig. 3. Scheme of finding the approximate profile.

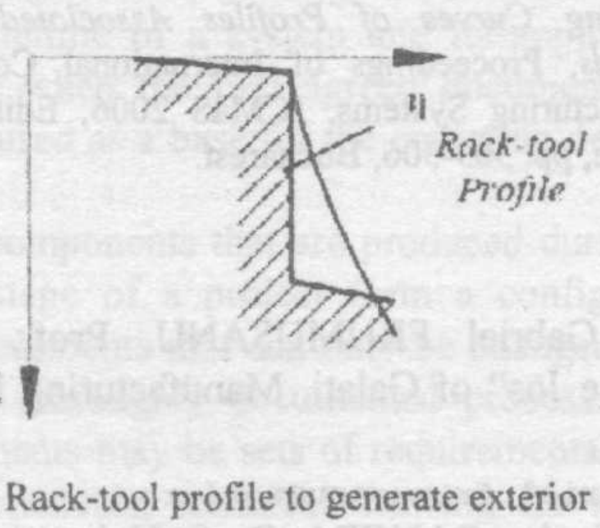

Fig. 4. Rack-tool profile to generate exterior slots.

Table 1

Rack-bar tool profiles (2nd degree approx. function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
4.9726	7.0319	4.9726	7.0319
4.8763	6.9768	4.8631	6.9728
4.7798	6.9221	4.7545	6.9145
4.6835	6.8681	4.6468	6.8571
4.5869	6.8145	4.5399	6.8006
4.4902	6.7614	4.4340	6.7449
4.3933	6.7088	4.3289	6.6900
4.2967	6.6570	4.2247	6.6360
4.1998	6.6056	4.1214	6.5829
4.1030	6.5548	4.0189	6.5306
4.0060	6.5045	3.9174	6.4792
…	…	…	…
-0.0346	4.9999	-0.0346	4.9999

Table 2

Rack-bar tool profiles (3rd degree approx. function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
4.9726	7.0319	4.9726	7.0319
4.8763	6.9768	4.8857	6.9782
4.7798	6.9221	4.7973	6.9247
4.6835	6.8681	4.7074	6.8716
4.5869	6.8145	4.6160	6.8188
4.4902	6.7614	4.5234	6.7663
4.3933	6.7088	4.4295	6.7142
4.2967	6.6570	4.3345	6.6625
4.1998	6.6056	4.2385	6.6112
4.1030	6.5548	4.1415	6.5603
4.0060	6.5045	4.0436	6.5099
3.9090	6.4548	3.9449	6.4599
…	…	…	…
-0.0346	4.9999	-0.0346	4.9999

Table 3

Rack-bar tool profiles (4th degree approx. function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
4.9726	7.0319	4.9726	7.0319
4.8763	6.9768	4.8691	6.9760
4.7798	6.9221	4.7672	6.9209
4.6835	6.8681	4.6669	6.8664
4.5869	6.8145	4.5679	6.8126
4.4902	6.7614	4.4700	6.7595
4.3933	6.7088	4.3731	6.7070
4.2967	6.6570	4.2770	6.6551
4.1998	6.6056	4.1816	6.6038
4.1030	6.5548	4.0866	6.5532
4.0060	6.5045	3.9920	6.5031
3.9090	6.4548	3.8977	6.4537
…	…	…	…
-0.0346	4.9999	-0.0346	4.9999

3.2. Rack-bar tool to generate arc of circle exterior elementary profiles

The case of an arc of circle exterior profile is now considered (Fig. 5). The input data were: $X_0 = -50$ mm; $Y_0 = 0$; $r = 10$ mm; $\hat{O} = 30°$; $R_{rp} = 60$ mm.

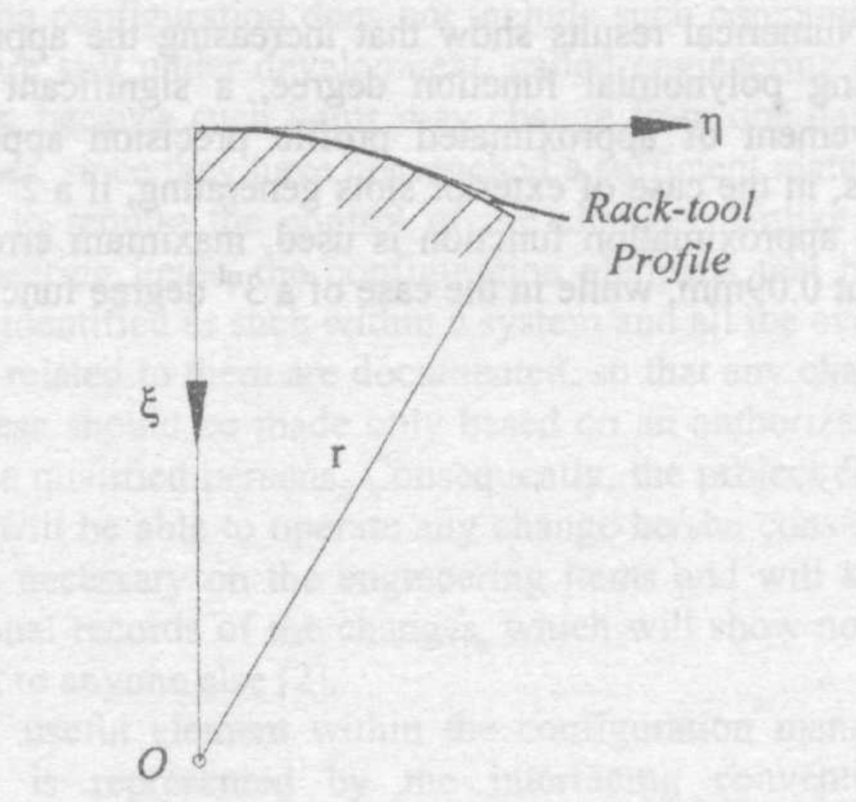

Fig. 5. Rack-tool profile to generate an exterior arc of circle.

There have also been used, successively, second and fourth degree functions to approximate the theoretical profile. The results are shown through points co-ordinates, respective, in Tables 4 and 5, near the theoretical profile points co-ordinates.

Table 4

Rack-bar tool profiles (2nd degree approx. function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-10.0000	-0.0104	-10.0000	-0.0104
-9.9993	0.0640	-10.0010	0.0635
-9.9979	0.1386	-10.0011	0.1378
-9.9959	0.2135	-10.0005	0.2125
-9.9931	0.2888	-9.9989	0.2875
-9.9897	0.3646	-9.9966	0.3630
-9.9855	0.4403	-9.9934	0.4389
-9.9807	0.5169	-9.9894	0.5151
-9.9751	0.5936	-9.9846	0.5918
-9.9689	0.6706	-9.9789	0.6688
-9.9619	0.7480	-9.9724	0.7462
...	...	...	...
-8.5842	5.1208	-8.5842	5.1208

Table 5

Rack-bar tool profiles (4th degree approx. function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-10.0000	-0.0104	-10.0000	-0.0104
-9.9993	0.0640	-9.9993	0.0639
-9.9979	0.1386	-9.9980	0.1385
-9.9959	0.2135	-9.9959	0.2134
-9.9931	0.2888	-9.9932	0.2887
-9.9897	0.3646	-9.9897	0.3644
-9.9855	0.4403	-9.9856	0.4404
-9.9807	0.5169	-9.9807	0.5167
-9.9751	0.5936	-9.9752	0.5935
-9.9689	0.6706	-9.9689	0.6705
-9.9619	0.7480	-9.9619	0.7480
...	...	...	...
-8.5842	5.1208	-8.5842	5.1208

4. CONCLUSIONS

Numerical results show that increasing the approximating polynomial function degree, a significant improvement of approximated profile precision appears. Thus, in the case of exterior slots generating, if a 2nd degree approximation function is used, maximum error is about 0.09mm, while in the case of a 3rd degree function, maximum error is only about 0.04 mm; best results were found by using a 4th degree function, maximum error decreasing to 0.02 mm.

The same conclusions can also be drawn in the case of generating an exterior arc of circle.

But using a superior degree approximation function do not bring only advantages; obviously, to use a 4th degree function also means that the co-ordinates of 5 points owning to the theoretical profile must be known to find polynomial coefficients, instead of 3 points (in the case of a 2nd degree function).

So, before choosing the degree of desired approximation function we must balance attentively between advantages and disadvantages.

REFERENCES

[1] Litvin, F. L. (1989). *Theory of Gearing*. NASA, Reference Publication, Washington DC, 1989.

[2] Favrolles, J. P. (1998). *Les surfaces complexes* (The complex surfaces).

[3] Oancea, N. (2004). *Generarea suprefețelor prin înfășurare, Vol. I - Teoreme fundamentale* (Surface generation by wrapping, Vol. I – Fundamental theorems), Edit. Fundației universitare „Dunărea de Jos", Galați.

[4] Oancea, N. (2004). *Generarea suprefețelor prin înfășurare, Vol.II - Teoreme complementare* (Surface generation by wrapping, Vol. II - Complementary theorems), Editura Fundației universitare "Dunărea de Jos", Galați.

[5] Oancea, N., Frumuşanu, G., Dura, G. (2006). *Algorithms for Representation by Poles as a Way to Approximate Wrapping Curves of Profiles Associated to Rolling Centrod*, Proceedings of International Conference on Manufacturing Systems, ICMaS 2006, Edit. Academiei Române, pp. 319-322, Bucharest.

[6] Frumuşanu, G., Oancea, N., Dura, G. (2006). *Applications of Representation by Poles as a Way to Approximate Wrapping Curves of Profiles Associated to Rolling Centrods*, Proceedings of International Conference on Manufacturing Systems, ICMaS 2006, Edit. Academiei Române, pp. 303-306, Bucharest.

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