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RACK-GEAR TOOL APPROXIMATIVE PROFILING BY BEZIER POLYNOMIALS

I. CIRCLE ARC ELEMENTARY PROFILE

BY

NICOLAE OANCEA*, IONUT POPA and VIRGIL TEODOR***

Abstract. In this paper is presented an algorithm for rack-gear tool's profiling, based on Bezier polynomials for the circle arc profiles. The approximate method, use a small points number on the profile (3 or 4 points regarding the used polynomial type). They are presented numerical examples which proof the method's quality, regarding the algorithms based on the surface enveloping fundamentals theorems.

Key words: surface enveloping, approximate profiling, Bezier polynomials, rack-gear profiling.

2000 Mathematics Subject Classification: 53B25, 53C15

1. Introduction

The rack-gear tool's profiling, reciprocally enveloping with an ordered surfaces curl, may be realized using representation by poles (Bezier polynomials) [1], [8], [9].

The rack-gear tool enveloping with an ordered surfaces curl, associated with a rolling centrodes couple may be determined using the fundamentals theorems of the surfaces enveloping [2], [6], as so as using the complementary methods [5], [7], [10]. Not in all situations meet in the industrial practice is need a very rigorous profiling of rack-gear tool's teeth flank. The worm cutter tool's form for generating the large side mill teeth, tools for spline shafts generation, allow to know not very rigorous the tool's profile.

In following is proposed a new approach of the rack-gear tool's profiling based on the method to approximate by Bezier polynomials the tool's profile.

The method presume to known a limited number of points (3 or 4, regarding the Bezier polynomial degree) which simplify the calculus and the time to profile.

The method presume a tabular algorithm, regarding the elementary profile to be generated: circle arc; involute arc; trochoidal arc, as simple profiles, which compose more complexes profiles, meet in industrial practice.

2. Circle arc elementary profile — Algorithm

In following, will be examinee the application for rack-gear tool's profile approximation methodology in case of a circle arc profile generation, see figure 1.

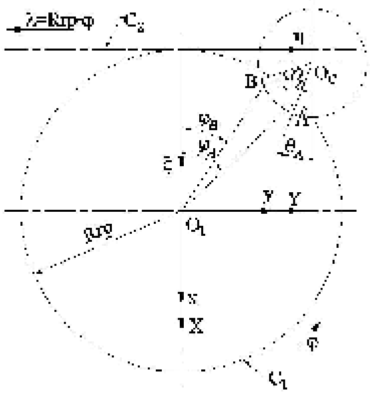

Fig. 1- Circular profile associated with the rolling centodes couple

Is defined the circle arc knowing two points on it, $A[X_A, Y_A]$ and $B[X_B, Y_B]$ as so as the value of *r* radius. Obviously, these are enough to determine the circle center $O_C[X_C, Y_C]$.

Are defined the circle's parametrical equations (the primary profile to be generated):

$$C\left|\begin{array}{l} X(q) = X_{O_C} + r \cdot \cos q; \\ Y(q) = Y_{O_C} - r \cdot \sin q, \end{array}\right. \quad (1)$$

q — variable parameter, between limits,

(2) $$q_A = \arccos\left[\frac{|X_A| - |X_{O_C}|}{r}\right] \text{ and } q_B = \arccos\left[\frac{|X_B| - |X_{O_C}|}{r}\right].$$

Table 1.

θ	**Primary profile**	**Enwrapping condition**
θ_A	$X_A = X_{O_C} + r \cdot \cos q_A$ $Y_A = Y_{O_C} - r \cdot \sin q_A$	$j_A = \arcsin\left[\frac{X_A \sin q_A + Y_A \cos q_A}{Rrp}\right]$
θ_C	$q_C = q_A + \frac{q_B - q_A}{2}$ $X_A = X_{O_C} + r \cdot \cos q_C$ $Y_A = Y_{O_C} - r \cdot \sin q_C$	$j_C = \arcsin\left[\frac{X_C \sin q_C + Y_C \cos q_C}{Rrp}\right]$
θ_B	$X_B = X_{O_C} + r \cdot \cos q_B$ $Y_B = Y_{O_C} - r \cdot \sin q_B$	$j_B = \arcsin\left[\frac{X_B \sin q_B + Y_B \cos q_B}{Rrp}\right]$
λ	**Points on the rack-gear profile**	**Approximation polynomial coefficients**
1	$x_A = X_A \cos j_A - Y_A \sin j_A + Rrp$ $h_A = X_A \sin j_A + Y_A \cos j_A + Rrp$	$A_x = x_A$ $A_h = h_A$
0.5	$x_C = X_C \cos j_C - Y_C \sin j_C + Rrp$ $h_C = X_C \sin j_C + Y_C \cos j_C + Rrp$	$B_x = \frac{x_C - 0.25 \cdot x_A - 0.25 \cdot x_B}{0.5}$ $B_h = \frac{h_C - 0.25 \cdot h_A - 0.25 \cdot h_B}{0.5}$
0	$x_B = X_B \cos j_B - Y_B \sin j_B + Rrp$ $h_B = X_B \sin j_B + Y_B \cos j_B + Rrp$	$C_x = x_B$ $C_h = h_B$

Is defined the circle arc family in the rolling motion of the two centrodes:

(3) $$(C)_j \left| \begin{aligned} x &= [X_{O_C} + r \cdot \cos q]\cos j - [Y_{O_C} - r \cdot \sin q]\sin j + Rrp; \\ h &= [X_{O_C} + r \cdot \cos q]\sin j + [Y_{O_C} - r \cdot \sin q]\cos j + Rrp \cdot j. \end{aligned} \right.$$

Table 2.

θ	Primary profile	Enwrapping condition
θ_A	$X_A = X_{O_C} + r \cdot \cos q_A$ $Y_A = Y_{O_C} - r \cdot \sin q_A$	$j_A = \arcsin\left[\dfrac{X_A \sin q_A + Y_A \cos q_A}{Rrp}\right]$
θ_C	$q_C = q_A + \dfrac{q_B - q_A}{3}$ $X_C = X_{O_C} + r \cdot \cos q_C$ $Y_C = Y_{O_C} - r \cdot \sin q_C$	$j_C = \arcsin\left[\dfrac{X_C \sin q_C + Y_C \cos q_C}{Rrp}\right]$
θ_D	$q_D = q_A + 2\dfrac{q_B - q_A}{3}$ $X_D = X_{O_C} + r \cdot \cos q_D$ $Y_D = Y_{O_C} - r \cdot \sin q_D$	$j_D = \arcsin\left[\dfrac{X_D \sin q_D + Y_D \cos q_D}{Rrp}\right]$
θ_B	$X_B = X_{O_C} + r \cdot \cos q_B$ $Y_B = Y_{O_C} - r \cdot \sin q_B$	$j_B = \arcsin\left[\dfrac{X_B \sin q_B + Y_B \cos q_B}{Rrp}\right]$
λ	**Points on the rack-gear profile**	**Approximation polynomial coefficients**
0	$x_A = X_A \cos j_A - Y_A \sin j_A + Rrp$ $h_A = X_A \sin j_A + Y_A \cos j_A + Rrp$	$D_x = x_A$ $D_h = h_A$
1/3	$x_C = X_C \cos j_C - Y_C \sin j_C + Rrp$ $h_C = X_C \sin j_C + Y_C \cos j_C + Rrp$	$C_x = \dfrac{18 \cdot x_C - 9 \cdot x_D + 2 \cdot x_B - 5 \cdot x_A}{6}$ $C_h = \dfrac{18 \cdot h_C - 9 \cdot h_D + 2 \cdot h_B - 5 \cdot h_A}{6}$
2/3	$x_D = X_D \cos j_D - Y_D \sin j_D + Rrp$ $h_D = X_D \sin j_D + Y_D \cos j_D + Rrp$	$B_x = \dfrac{-5 \cdot x_B + 2 \cdot x_A + 18 \cdot x_D - 9 \cdot x_C}{6}$ $B_h = \dfrac{-5 \cdot h_B + 2 \cdot h_A + 18 \cdot h_D - 9 \cdot h_C}{6}$
1	$x_B = X_B \cos j_B - Y_B \sin j_B + Rrp$ $h_B = X_B \sin j_B + Y_B \cos j_B + Rrp$	$A_x = x_B$ $A_h = h_B$

Similarly, are defined the values of the j angular parameter, in the rolling motion of the two centrodes, knowing the normal at circle in the current point,

$$\overrightarrow{N_C}:[X-X(q)]\sin q+[Y-Y(q)]\cos q=0 \tag{4}$$

and the C_1 centrode equations:

$$C_1\left|\begin{array}{l} X=Rrp\cdot\cos j\,; \\ Y=Rrp\cdot\sin j\,. \end{array}\right. \tag{5}$$

From these, result the conditions to determine the j angle value, corresponding to the characteristically points on the circle,

$$j=\arcsin\left[\frac{X(q)\sin q+Y(q)\cos q}{Rrp}\right]-q\,, \tag{6}$$

condition which represents the specifically enwrapping condition, see table 1, for a approximation polynomial of 2nd degree and, similarly, in table 2, for a polynomial of 3rd degree.

In tables 1 and 2, they are presented the calculus elements of the Bezier approximation polynomials for tool profile associated with the rack-gear.

The Bezier polynomial for 2nd degree is:

$$\begin{aligned} x&=l^2A_x+2l(1-l)B_x+(1-l)^2C_x\,; \\ h&=l^2A_h+2l(1-l)B_h+(1-l)^2C_h\,, \end{aligned} \tag{7}$$

and for 3rd degree is:

$$\begin{aligned} x&=l^3A_x+3l^2(1-l)B_x+3l(1-l)^2C_x+(1-l)^3D_x\,; \\ h&=l^3A_h+3l^2(1-l)B_h+3l(1-l)^2C_h+(1-l)^3D_x\,. \end{aligned} \tag{8}$$

3. Numerical examples

Based on the presented algorithm, is showed a numerical example for a circular profile, see figure 1, for rack-gear tool's profiling, reciprocally enveloping with an circle arc defined between points $A[-87.02;4.8]$ and $B[-80;20]$, with characteristics: $r=20$ mm; center coordinates $O_C[-100,20]$, profile associated with a circular centrode with radius $R_{rp}=100$ mm.

In tables 3 and 4 and figure 2, are presented the profile form and coordinates for approximation polynomial of 2nd and 3rd degree, regarding the profile calculated by one of the fundamentals methods [2], [7], [10].

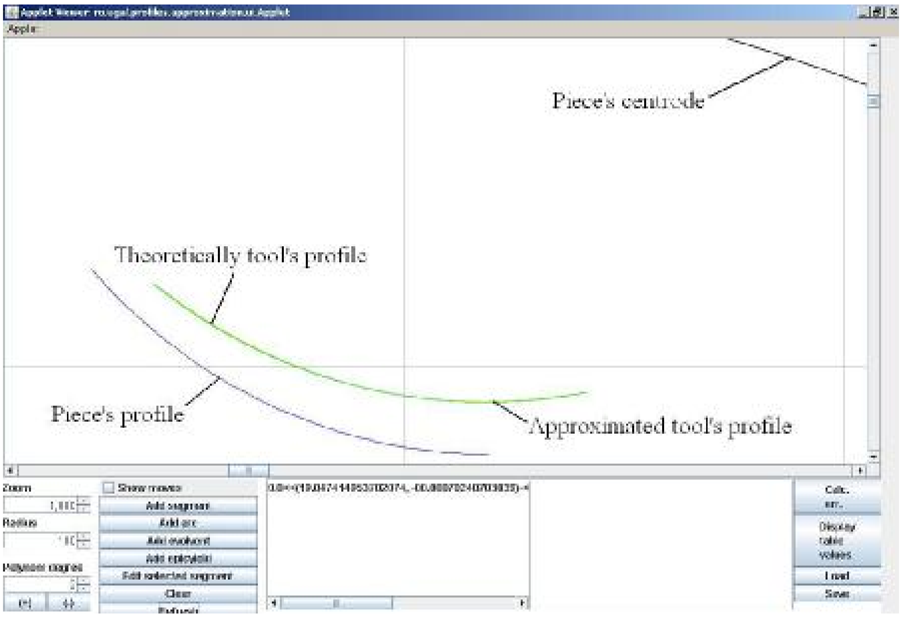

Fig. 2– Approximated and theoretically rack-gear profile

Table 3.

l	Approximated tool profile		Theoretical tool profile		Error [mm]	j [rad]
	x [mm]	h [mm]	x [mm]	h [mm]		
0.0	**13.5372**	**7.1565**	**13.5372**	**7.1565**	**0.0000**	**0.1810**
	14.0700	7.8659	14.0867	7.8634	0.0169	0.1832
	⋮	⋮	⋮	⋮	⋮	⋮
	17.1984	14.1061	17.2013	14.1085	0.0038	0.1926
0.5	**17.4269**	**14.9399**	**17.4269**	**14.9399**	**0.0000**	**0.1929**
	17.6172	15.7827	17.6166	15.7800	0.0028	0.1932
	⋮	⋮	⋮	⋮	⋮	⋮
	17.7823	22.7256	17.7944	22.7231	0.0123	0.2005
1.0	**17.6438**	**23.5943**	**17.6438**	**23.5943**	**0.0000**	**0.2020**

Table 4.

l	**Approximated tool profile**		**Theoretical tool profile**		**Error** [mm]	j [rad]
	x [mm]	h [mm]	x [mm]	h [mm]		
0.0	**13.5372**	**7.1565**	**13.5372**	**7.1565**	**0.0000**	**0.1810**
	14.0846	7.8609	14.0867	7.8634	0.0033	0.1832
	M	M	M	M	M	M
	16.2983	11.6531	16.2959	11.6478	0.0059	0.1895
0.333	**16.5243**	**12.1824**	**16.5243**	**12.1824**	**0.0000**	**0.1904**
	16.6283	12.4410	16.6304	12.4468	0.0062	0.1900
	M	M	M	M	M	M
	17.8906	17.5162	17.8900	17.5123	0.0040	0.1955
0.666	**17.9181**	**17.7753**	**17.9181**	**17.7753**	**0.0000**	**0.1950**
	17.9679	18.3641	17.9684	18.3727	0.0086	0.1959
	M	M	M	M	M	M
	17.7943	22.7240	17.7944	22.7231	0.0009	0.2005
1.0	**17.6438**	**23.5943**	**17.6438**	**23.5943**	**0.0000**	**0.2020**

In following is showed another numerical example for a circular profile, for rack-gear tool reciprocally enveloping with an circle arc defined between points $A[-95;-10]$ and $B[-105;5]$, with characteristics: $r=15$ mm; profile associated with a circular centrode with radius $R_{rp}=100$ mm.

4. Conclusions

1. The presented method, although approximately, assure a good representation of the rack-gear tool's profile reciprocally enveloping with a circular profile, which may be part of a composed profile to be generated.
2. The method is simple, used a limited points number on the profile to be generated, so the calculus time is reduced.
3. Software dedicated to this algorithm is an instrument which helps to apply this method.
4. In order to increase the profiling precision is necessary to increase the Bezier polynomials degree.

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**"Dunărea de Jos" University of Galaţi*
Manufacturing Science and Engineering Department
***"Al. I. Cuza" University of Iaşi*
Faculty of Computer Science

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Technical University "Gheorghe Asachi"
Department of Machine Manufacturing Technology
Iaşi, Romanin,
e-mail: tmcriasi@gmail.com

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PROFILAREA APROXIMATIVĂ PRIN POLINOAME BEZIER A SCULEI-CREMALIERĂ

(Rezumat)

În lucrare, se prezintă un algoritm de profilare a sculei-cremalieră, în baza reprezentării prin polinoame Bezier, destinat generării prin înfăşurare prin metoda rulării a profilurilor în arc de cerc.

Metoda aproximativă, utilizează un număr redus de puncte de pe profil (3 sau 4 puncte în funcţie de gradul polinomului utilizat).

Sunt prezentate exemple numerice care atestă calitatea metodei, în comparaţie cu algoritmi bazaţi pe teoremele fundamentale ale înfăşurării suprafeţelor.