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ALGORITHMS IMPROVING REPRESENTATION BY POLES WHEN GENERATING BY TOOLS ASSOCIATED TO CIRCULAR CENTRODS

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Abstract: *In this paper, the algorithms used to improve the representation by poles precision in the case of profiles from whirls of surfaces generated by rolling are adapted and used to profile tools associated to circular centrods: the pinion cutter and the rotating cutter to generate worm's axial sections. Numerical examples of applying the improved representation algorithms in some concrete cases are also presented and commented.*

Key words: *representation by poles, enwrapped surfaces, pinion cutter, rotating cutter.*

1. INTRODUCTION

Algorithms conceived to improve profiles representation by poles precision in the case of generating by using rack-tools and their application were already presented [6].

We shall further analyze the specific problems concerning approximation precision when representing by poles the enwrapped of profiles associated to a couple of rolling centrods, if pinion-cutters or rotating cutters are used to generate.

Similar, the problem of finding the degree of polynomial function to approximate by poles the enwrapping curves requires to be solved when profiling tools generating by rolling, associated to circular centrods, the cases of pinion cutter and rotating cutter, specific algorithms being established [5, 6].

Based on general laws used to study enwrapped surfaces [1], when expressing enwrapping curves by poles [2], numeric applications were developed using the suggested algorithms [5, 6] to profile tools as the pinion cutter or the rotating cutter.

2. THE CASE OF GENERATING BY USING A PINION CUTTER

In the case of generating by using pinion-cutter type of tools, Fig. 1, the following reference systems must be considered:

- xyz, as a global system, having the rotation axis coincident to the axis of the profile to be generated (Σ) centrod (C_1);
- $x_0y_0z_0$ - global system, having the same rotation axis as the pinion-cutter (C_2);
- XYZ - relative system, joint to C_1 centrod;
- $\xi\eta\zeta$ - relative system, joint to the pinion-cutter.

Generating process kinematics, outgoing from the two circular centrods rolling motion, according to condition R

$$R_{rp}\cdot\varphi_1 = R_{rs}\cdot\varphi_2, \tag{1}$$

is described through the following relative motions:

$$\xi = \omega_3(-\varphi_2)\left[\omega_3^T(\varphi_1)\cdot X - A\right], \tag{2}$$

with

$$A = \|-A_{12} \quad 0 \quad 0\|^T \tag{3}$$

and its reversed

$$X = \omega_3(\varphi_1)\left[\omega_3^T(-\varphi_2)\cdot\xi + A\right], \tag{4}$$

as representing the relative motions between the spaces associated to the two rolling centrods.

To a certain profile, Σ, represented in polar form as

$$\Sigma = \left\| \begin{matrix} P_X(\lambda) \\ P_Y(\lambda) \end{matrix} \right\|, \tag{5}$$

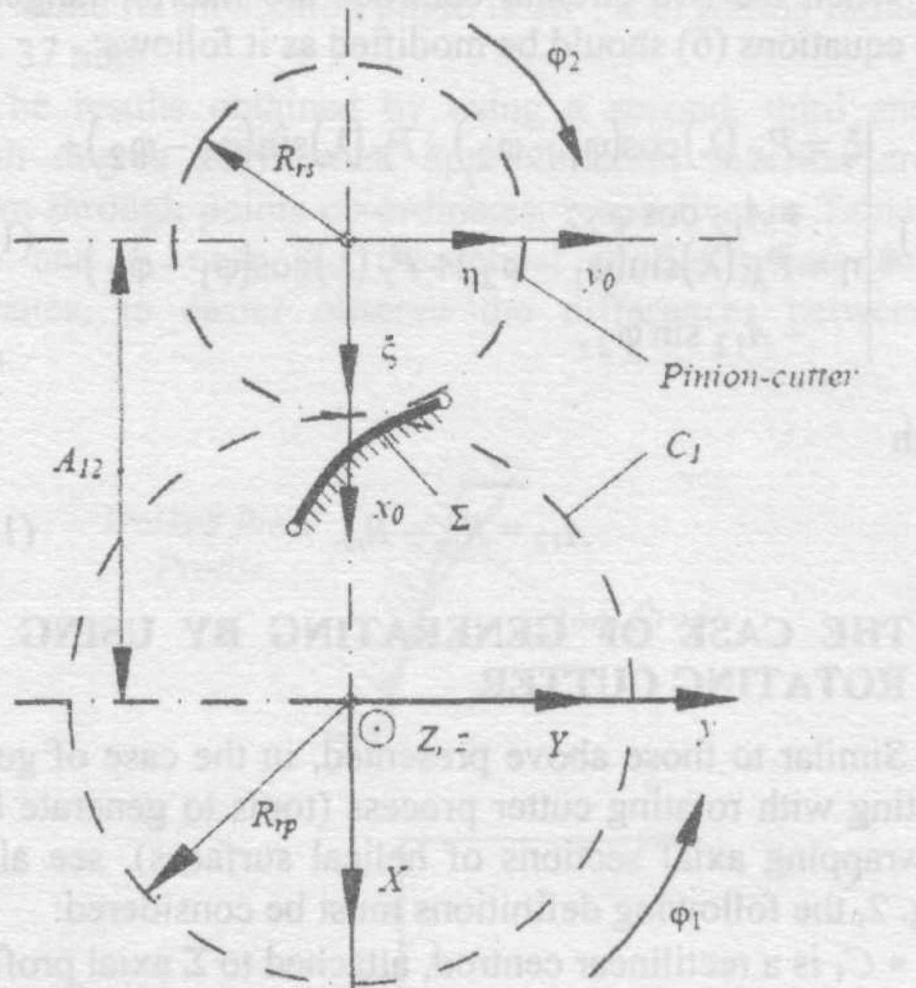

Fig.1. Pinion-cutter generating scheme.

with λ – variable parameter, from relation (2) the family of profiles results in the shape:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = P_X(\lambda)\cos\varphi - P_Y(\lambda)\sin\varphi + A_{12}\cos\varphi_2; \\ \eta = P_X(\lambda)\sin\varphi + P_Y(\lambda)\cos\varphi - A_{12}\sin\varphi_2, \end{vmatrix} \quad (6)$$

where $\varphi = \varphi_1 + \varphi_2$, $i = \dfrac{\varphi_1}{\varphi_2}$ and $A_{12} = R_{rp} + R_{rs}$. (7)

Usually i (meaning the transmission ratio) is a constant. The enwrapped of $(\Sigma)_\varphi$ profiles family, (6), results by associating to these equations the enveloping condition written by using Perpendiculars Method (Willis) [1, 3] which, by considering anterior notations, the definition of the normal to Σ surface, referred to XYZ system,

$$N_\Sigma : [X - P_X(\lambda)]P'_{X_\lambda} + [Y - P_Y(\lambda)]P'_{Y_\lambda} = 0 \quad (8)$$

and also the equations of C_1 centrod

$$C_1 : \begin{vmatrix} X = -R_{rp}\cos\varphi_1; \\ Y = R_{rp}\sin\varphi_1, \end{vmatrix} \quad (9)$$

from the condition of intersection between the two curves, N_Σ and C_1, results as

$$-P'_{X_\lambda}\cos\varphi_1 + P'_{Y_\lambda}\sin\varphi_1 = \frac{P_X \cdot P'_{X_\lambda} + P_Y \cdot P'_{Y_\lambda}}{R_{rp}}. \quad (10)$$

Finally, by varying φ, pinion-cutter profile results as reciprocal enwrapped to Σ profile, numerically expressed through a matrix like

$$S = \begin{Vmatrix} \xi_i \\ \eta_i \end{Vmatrix}^T (i = 1 \ldots n). \quad (11)$$

When the two circular centrods are interior tangent, the equations (6) should be modified as it follows:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = P_X(\lambda)\cos(\varphi_1 - \varphi_2) - P_Y(\lambda)\sin(\varphi_1 - \varphi_2) + \\ \quad + A_{12}\cos\varphi_2; \\ \eta = P_X(\lambda)\sin(\varphi_1 - \varphi_2) + P_Y(\lambda)\cos(\varphi_1 - \varphi_2) - \\ \quad - A_{12}\sin\varphi_2, \end{vmatrix} \quad (12)$$

with

$$A_{12} = R_{rp} - R_{rs}. \quad (13)$$

3. THE CASE OF GENERATING BY USING A ROTATING CUTTER

Similar to those above presented, in the case of generating with rotating cutter process (tools to generate by enwrapping axial sections of helical surfaces), see also Fig. 2, the following definitions must be considered:

- C_1 is a rectilinear centrod, attached to Σ axial profile of helical surface to be generated;
- C_2 – circular centrod, attached to the rotating cutter;
- xyz – global system, having Oz axis overlaid to rotating cutter rotation axis;

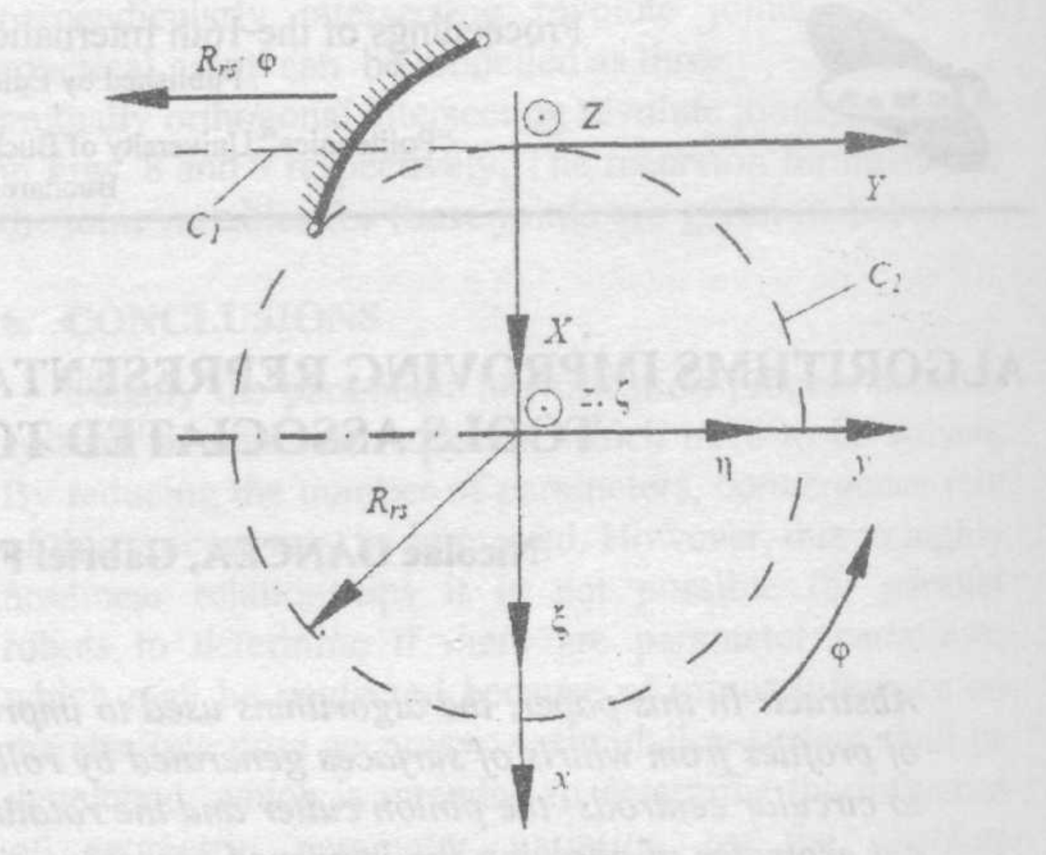

Fig.2. Rotating cutter generating scheme.

- XYZ and $\xi\eta\zeta$ – relative systems, attached to the two rolling centrods, during their motions.

In the rolling process between the two centrods, the following relative motions can be expressed:

$$\xi = \omega_3(\varphi)[X + a] \quad (14)$$

and its reverse

$$X = \omega_3^T(\varphi) \cdot \xi - a \quad (15)$$

where

$$a = \begin{Vmatrix} -R_{rs} \\ R_{rs} \cdot \varphi \end{Vmatrix}, \quad (16)$$

with R_{rs} meaning rotating cutter centrod radius.

If the axial profile of helical surface to be generated is given by a relation of (5) type, the family of profiles can be found, referred to $\xi\eta\zeta$ system, as:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = [P_X(\lambda) - R_{rs}]\cos\varphi + [P_Y(\lambda) - R_{rs} \cdot \varphi]\sin\varphi; \\ \eta = -[P_X(\lambda) - R_{rs}]\sin\varphi + [P_Y(\lambda) - R_{rs} \cdot \varphi]\cos\varphi. \end{vmatrix} \quad (17)$$

The enveloping condition associated to the family of profiles (17), specific to "Minimum Distance Method" [4] is

$$[\xi(\lambda,\varphi) + R_{rs}\cos\varphi] \cdot \xi'_\lambda + [\eta(\lambda,\varphi) - R_{rs}\sin\varphi] \cdot \eta'_\lambda = 0, \quad (18)$$

where $\xi(\lambda, \varphi)$ and $\eta(\lambda, \varphi)$ have the significance from relation (17).

Finally, by varying the parameter remaining in (17) after using (18), rotating cutter profile results expressed through a matrix similar to the one shown by (11).

4. NUMERICAL APPLICATIONS

To observe the effect of approximation polynomial function degree increasing, second, third and fourth degree polynomial functions were successively used to approximate the same pinion-cutter theoretical profile, respective the same rotating cutter theoretical profile.

4.1. Pinion-Cutter Profile Approximation

In this paragraph, the case of an interior triangular profile, generated by using a pinion cutter is exemplified (Fig. 3). By using profile symmetry, only one of two profile flanks was considered.

Table 1

Pinion-Cutter Profiles (2nd Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-40.6672	-4.5286	-40.6672	-4.5286
-40.8113	-4.4753	-40.8102	-4.4779
-40.9559	-4.4214	-40.9536	-4.4264
-41.1009	-4.3669	-41.0975	-4.3740
-41.2463	-4.3116	-41.2417	-4.3208
-41.3918	-4.2559	-41.3864	-4.2667
-41.5377	-4.1995	-41.5315	-4.2118
-41.6837	-4.1426	-41.6771	-4.1561
-41.8305	-4.0848	-41.8231	-4.0995
-41.9774	-4.0266	-41.9695	-4.0421
-42.1243	-3.9677	-42.1163	-3.9838
-42.2717	-3.9082	-42.2635	-9.9247
...	...	...	...
-50.0000	0.0000	-50.0000	0.0000

Table 2

Pinion-Cutter Profiles (3rd Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-40.6672	-4.5286	-40.6672	-4.5286
-40.8113	-4.4753	-40.8117	-4.4749
-40.9559	-4.4214	-40.9565	-4.4206
-41.1009	-4.3669	-41.1016	-4.3658
-41.2463	-4.3116	-41.2469	-4.3105
-41.3918	-4.2559	-41.3925	-4.2546
-41.5377	-4.1995	-41.5384	-4.1981
-41.6837	-4.1426	-41.6846	-4.1411
-41.8305	-4.0848	-41.8312	-4.0834
-41.9774	-4.0266	-41.9780	-4.0252
-42.1243	-3.9677	-42.1251	-3.9663
-42.2717	-3.9082	-42.2725	-3.9068
...	...	...	...
-50.0000	0.0000	-50.0000	0.0000

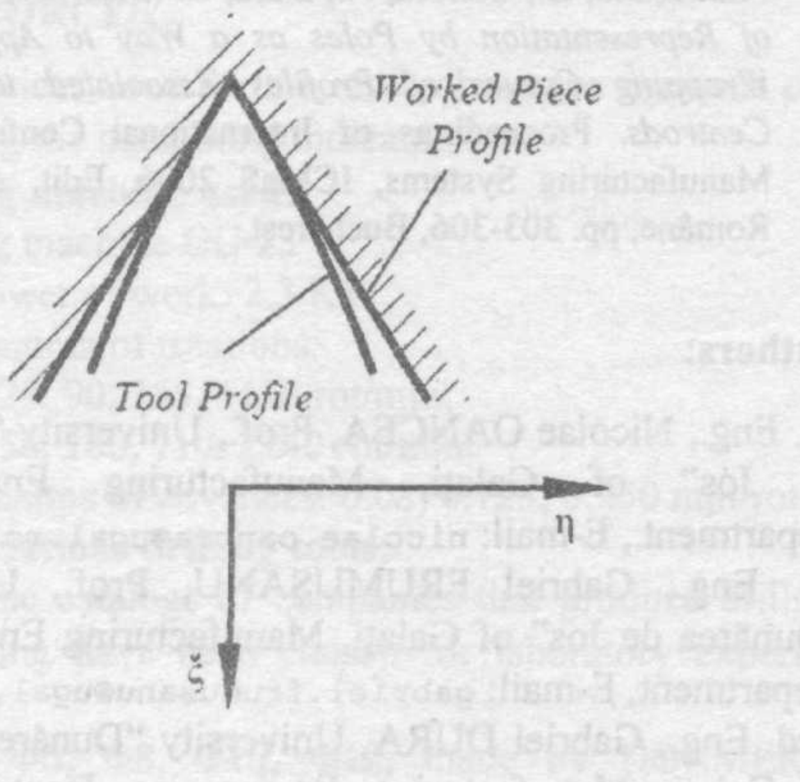

Fig.3. Pinion-Cutter Profile.

Table 3

Pinion-Cutter Profiles (4th Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-40.6672	-4.5286	-40.6672	-4.5286
-40.8113	-4.4753	-40.8114	-4.4753
-40.9559	-4.4214	-40.9560	-4.4215
-41.1009	-4.3669	-41.1009	-4.3670
-41.2463	-4.3116	-41.2461	-4.3119
-41.3918	-4.2559	-41.3916	-4.2561
-41.5377	-4.1995	-41.5375	-4.1997
-41.6837	-4.1426	-41.6837	-4.1427
-41.8305	-4.0848	-41.8302	-4.0851
-41.9774	-4.0266	-41.9770	-4.0268
-42.1243	-3.9677	-42.1242	-3.9678
-42.2717	-3.9082	-42.2717	-3.9083
...	...	...	...
-50.0000	0.0000	-50.0000	0.0000

The input data, used to run this application, concerning a pinion cutter profiling, were:

- the coordinates of straight line segment AB end-points, giving the profile to be generated, $X_A = -100$ mm; $Y_A = 0$; $X_B = -89.78$ mm; $Y_B = -6.28$ mm;
- segment AB inclination angle, referred to horizontal direction, $\hat{A} = 60°$;
- worked piece's rolling radius, $R_{rp} = 100$ mm;
- $i = 2$ – transmission ratio, see (7).

The results obtained by using a second, third and fourth degree polynomial approximation function are shown through points coordinates, respectively, in Tables 1, 2 and 3, near the theoretical profile points coordinates, to realize a better comparison.

4.2. Rotating Cutter Profile Approximation

The example considered is concerning a rotating cutter used to generate a worm with trapezoidal axial section (see Fig. 4). Worm profile height, in axial section, is 10 mm, while its inclination angle is 20°; tool rolling radius $R_{rs} = 37$ mm.

The results obtained by using a second, third and fourth degree polynomial approximation function are shown through points co-ordinates, respective, in Tables 4, 5 and 6, near the theoretical profile points co-ordinates, to easier observe the differences between them.

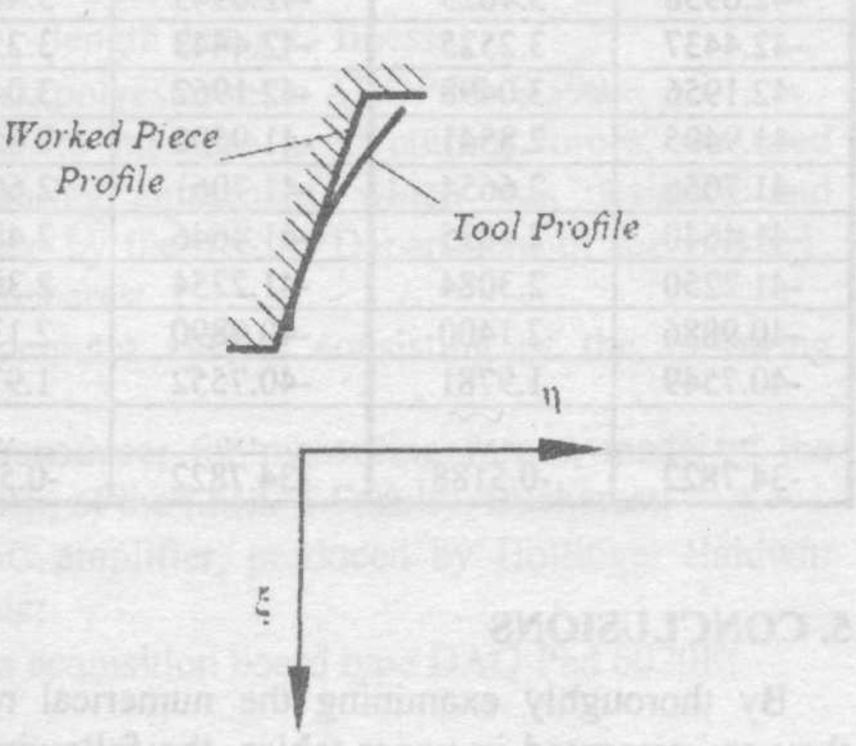

Fig.4. Rotating Cutter Profile.

Table 4

Rotating Cutter Profiles (2nd Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-43.4542	4.1354	-43.4542	4.1354
-43.1992	3.9037	-43.1708	3.9393
-42.9457	3.6794	-42.8921	3.7472
-42.6938	3.4623	-42.6181	3.5591
-42.4437	3.2525	-42.3489	3.3751
-42.1956	3.0498	-42.0843	3.1950
-41.9495	2.8541	-41.8244	3.0190
-41.7056	2.6654	-41.5693	2.8470
-41.4640	2.4835	-41.3189	2.6790
-41.2250	2.3084	-41.0731	2.5150
-40.9886	2.1400	-40.8321	2.3551
-40.7549	1.9781	-40.5958	2.1992
...	...	...	...
-34.7822	-0.5188	-34.7822	-0.5188

Table 5

Rotating Cutter Profiles (3rd Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-43.4542	4.1354	-43.4542	4.1354
-43.1992	3.9037	-43.1944	3.9006
-42.9457	3.6794	-42.9370	3.6738
-42.6938	3.4623	-42.6819	3.4548
-42.4437	3.2525	-42.4295	3.2435
-42.1956	3.0498	-42.1796	3.0398
-41.9495	2.8541	-41.9323	2.8435
-41.7056	2.6654	-41.6879	2.6545
-41.4640	2.4835	-41.4462	2.4726
-41.2250	2.3084	-41.2075	2.2978
-40.9886	2.1400	-40.9718	2.1299
...	...	...	...
-34.7822	-0.5188	-34.7822	-0.5188

Table 6

Rotating Cutter Profiles (4th Degree Approx. Function)

Theoretical Profile		Approximated Profile	
ξ [mm]	η [mm]	ξ [mm]	η [mm]
-43.4542	4.1354	-43.4542	4.1354
-43.1992	3.9037	-43.1994	3.9033
-42.9457	3.6794	-42.9461	3.6786
-42.6938	3.4623	-42.6943	3.4614
-42.4437	3.2525	-42.4443	3.2514
-42.1956	3.0498	-42.1962	3.0487
-41.9495	2.8541	-41.9500	2.8530
-41.7056	2.6654	-41.7061	2.6643
-41.4640	2.4835	-41.4646	2.4826
-41.2250	2.3084	-41.2254	2.3076
-40.9886	2.1400	-40.9890	2.1393
-40.7549	1.9781	-40.7552	1.9775
...	...	...	...
-34.7822	-0.5188	-34.7822	-0.5188

5. CONCLUSIONS

By thoroughly examining the numerical results as they are presented in upper tables, the following g conclusions can be drawn:

- In the case of the pinion-cutter used to generate an internal triangular profile, the maximum error which appears when approximating the theoretical profile, expressed by poles, through a second degree polynomial function is about $1\cdot10^{-2}$ mm. If a third degree polynomial function is used to realize the approximation, the maximum error decreases at about $1\cdot10^{-3}$ mm, while a fourth degree polynomial approximation function leads to a maximum error comparable to $1\cdot10^{-4}$ mm.
- In the case of the rotating cutter used to generate a worm with trapezoidal axial section, when the theoretical profile expressed by poles is approximated by a second degree polynomial function, the maximum error is about $1\cdot10^{-1}$ mm. If approximation function degree is increased at 3 or 4, the maximum error resulted is about $1\cdot10^{-2}$ mm, respective 10^{-3} mm.
- The approximation errors are proportional to the length of substituted profile.
- By considering the maximum errors acceptable from technical point of view, we don't need every time a superior degree approximation function; sometimes a second degree function is good enough and its application has the advantage of simplicity. We can also conclude that in almost all practical situations encountered, a third degree approximation function offers very good results.

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