Algorithm for Corrective Profiling of Rack-Gear Tools

PhD. eng. Virgil Teodor, PhD. eng. Marian Cucu, PhD. eng. Nicolae Oancea
"Dunărea de Jos" University of Galați

ABSTRACT

Is presented a mathematical model, starting from a virtual generatrix of the surface to be generated, and, based on this, an algorithm for corrective profiling of rack-gear tool, in order to increase the machining precision at generation by rolling of this tools type.

We accept that the surfaces to be generated, the target surfaces, are effective measured surfaces and are expressed in numerical form. Is imagined a model for the approximation of the corrected rack tool's profile by the back face form's changing regarding the standard form – the approximated profile.

It was elaborated a software for this model and are presented numerical examples.

Keywords: corrective profiling, rack-gear tool, virtual generatrix.

1. Introduction

The improvement of the machining precision of complex surfaces, generated by reciprocally enveloping surfaces principle, assume, more and more, the synthesis of new models for prediction and compensation by software of generating errors determined by on-machine measuring (OMM [4]), the whole system being CAD/CAM/CAI integrated: analytical models [1], [2], genetic algorithm based modeling [3], polynomial neural network based methods [4].

Were developed methods for errors compensation by programming tool's alternative path, assuring, by cutting forces decreasing, the decreasing of the generating errors [7], [8].

Also, based on the conjugated surfaces theory, were developed methods for discreetly known surfaces generation (digital gear tooth surfaces – DGTS) [9].

In this paper, for tools generating by enwrapping, by rolling method, is proposed a new algorithm for generated surface's correction by generating tool's profile correction, by a corrective sharpening of back face of rack-gear tool, which leads at an effective tool's profile which allows the form correction of the generated profile, in order to increase the generation precision of this.

2. Method principle

Regarding the specifically form of tools which generate by rolling method —the gear shaped tool, see figure 1, re-profiling of these, in order to correct the errors determined by measuring on the blank profile is difficult and usually less economic.

The tools machine with numerical command, which have OMM systems, allow to measure the effective surface and to express in discreetly form the generatrix of these. Having this profile as base, is possible to determine a new corrected tool's profile, which at machining resuming will generate on blank a new profile with a smaller error regarding the theoretically surface.

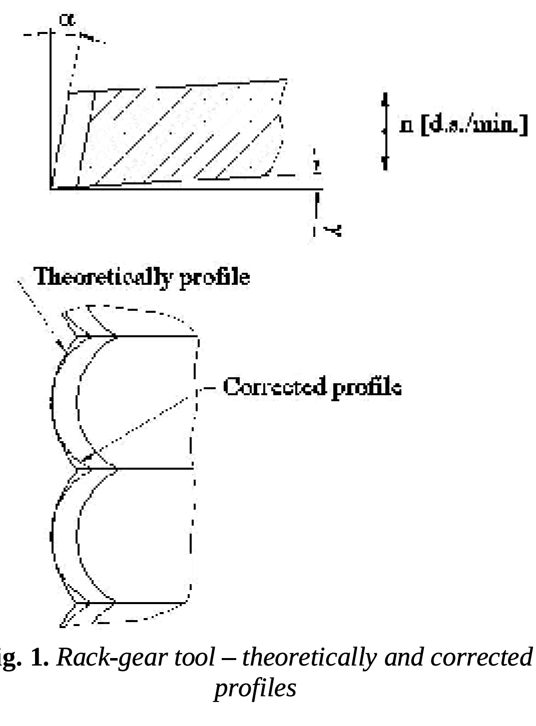

Fig. 1. *Rack-gear tool – theoretically and corrected profiles*

This approach of the surfaces generation assumes to solve two problems:

- to determine a virtual profile, as target profile different regarding the theoretically profile of the surface to be generated;
- to re-profile the tool, in order to correct its profile, regarding the new target profile, profiling which should be made starting from a measured profile and expressed in discreetly form.

The correction algorithm assume that the technological system will react in same way as at effective surface generating phase, phase accepted as base for the correction. In this way, establishing the target profile, we can consider that the machining with the corrected tool will lead to a new surface effective profile, closer as form and dimensions to the theoretically profile, for the same blank or, in case of a repeatable machining, for all of the machined pieces.

2.1. Theoretically, effective and virtual profiles

The effective generated profile may be represented by an array having as elements the point’s coordinates of these profiles, (fig 2).

Obviously, in mostly situations, the effective profile isn’t identically with the theoretically profile due of frequently unidentifiable causes.

$$\Sigma = \begin{Vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \\ \vdots & \vdots \\ Y_m & Y_m \end{Vmatrix}. \quad (1)$$

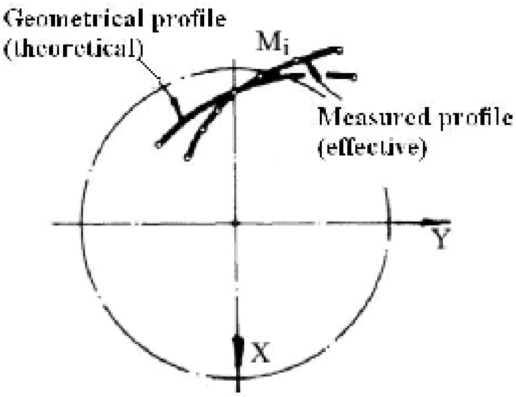

Fig. 2. *Theoretically and effective profiles*

The correction method proposed the notion of *virtual profile (generatrix)* obtained as the mirror of points belongs to effective profile, regarding the theoretically profile (fig. 3).

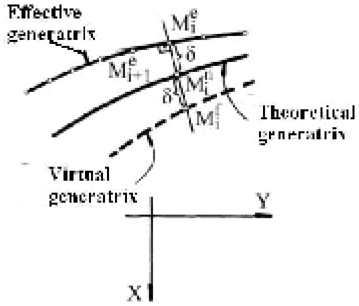

Fig. 3. *Theoretically, effective and virtual profiles (generatrix)*

The tool’s virtual profile construction is obtained in the following way:

- is obtained the normal at theoretically profile in the M_i point, knowing two closed points along this profile

$$M_i^e = \begin{Bmatrix} X_i^e \\ Y_i^e \end{Bmatrix} \quad (2)$$

$$M_{i+1}^e = \begin{Bmatrix} X_{i+1}^e \\ Y_{i+1}^e \end{Bmatrix}, \quad (3)$$

with condition

$$\sqrt{\left(X_i^e - X_{i+1}^e\right)^2 + \left(Y_i^e - Y_{i+1}^e\right)^2} < e \quad (4)$$

$e = 1\ldots 0.1mm$

with equation

$$(X - X_i^e)N_x + (Y - Y_i^e)N_y = 0 \quad (5)$$

where N_x and N_y are directix parameters of the normal, defining:

$$tga_i = \frac{\left|Y_{i+1}^e - Y_i^e\right|}{\left|X_{i+1}^e - X_i^e\right|};$$
$$N_x = \cos a_i; \quad (6)$$
$$N_y = \sin a_i.$$

The normal is intersected with the theoretically profile, known by the parametrical equations:

$$X = X(u); \quad Y = Y(u), \quad (7)$$

with u variable, obtaining the M_i^n point’s coordinates on this profile.

It’s calculated the δ distance between points M_i^e and M_i^n,

$$d = \sqrt{(X_i^n - X_i^e)^2 + (Y_i^n - Y_i^e)^2} \quad (8).$$

It’s determined the current point’s coordinates on the virtual profile,

$$M_i^f : \begin{cases} X_i^f = X_i^n + d\cos a_i; \\ Y_i^f = Y_i^n + d\sin a_i. \end{cases} \quad (9).$$

The M_i^f points assembly determine the virtual profile as base for the determination of the correct tool's profile.

3. Algorithm for corrective profiling of rack-gear tool

In following will be presented a specifically algorithm or corrective profiling of the rack-gear tool.

Knowing the laws or rack-toll profile determination (the peripheral primary surface), starting from the theoretically generatrix of the surface to be generate and defining the surface *virtual generatrix* as a new target surface is build the array

$$\Sigma_v = \begin{Vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \\ \vdots & \vdots \\ Y_m & Y_m \end{Vmatrix}, \qquad (10)$$

see figure 4.

Is proposed a new algorithm, based on the "tangents method" [10] applied for the virtual generatrix of the new target surface. So, the virtual generatrix family is :

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{Vmatrix} \cdot \begin{Vmatrix} X_i^F \\ Y_i^F \end{Vmatrix} - \begin{Vmatrix} -R_{rp} \\ -R_{rp}\cdot\varphi_1 \end{Vmatrix}, (i = 1, 2, ..., m). \qquad (11)$$

After development, result:

$$\begin{vmatrix} \xi = X_i^F \cdot \cos\varphi_1 - Y_i^F \cdot \sin\varphi_1 + R_{rp}; \\ \eta = X_i^F \cdot \sin\varphi_1 + Y_i^F \cdot \cos\varphi_1 + R_{rp} \cdot \varphi_1. \end{vmatrix} \qquad (12)$$

The (11) and (12) equations represent the virtual generatrix family model in the rack-gear reference system (see figure 4).

The specifically enwrapping condition may be bring at form

$$\left[Y_i^F - R_{rp}\cdot\sin\varphi_1\right]\cdot\sin\beta_i^F - \\ -\left[-X_i^F - R_{rp}\cdot\cos\varphi_1\right]\cdot\cos\beta_i^F = 0. \qquad (13)$$

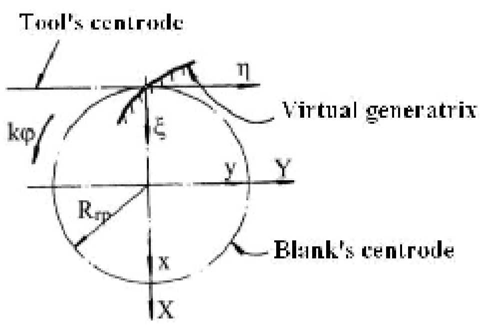

Fig. 4. *References systems*

In the previous equations, X_i^F, Y_i^F are coordinates of the points belongs of the virtual generatrix. The (12) and (13) equation assembly represent the rectified rack-gear tool profile.

In order to verify the proposed algorithm was used an algorithm developed based on the "minimum distance" method [11].

Is determined the virtual generatrix family in the tool's reference system in movement:

$$x = \omega_3^T(k\varphi)X - a \qquad (14)$$

with

$$a = \begin{Vmatrix} -R_{rp} \\ -R_{rp}k\varphi \end{Vmatrix} \text{ (k=1, 2, ..., n);} \qquad (15)$$

φ - angular increment of the revolution movement.

The virtual generatrix family is expressed by an array of type

$$(G)_{k\varphi} = \left\{ \begin{Vmatrix} x_1 & x_2 & \ldots & x_m \\ \eta_1 & \eta_2 & \ldots & \eta_m \end{Vmatrix} \right\}_{k\varphi} \qquad (16)$$

$$(k = 1, 2, ..., n).$$

The specifically enwrapping condition, associated with the $(G)_{k\varphi}$ family is:

$$d_{\min} = \sqrt{x_i^2 + (\eta_i - R_{rp}k\varphi)^2}, \\ (i = 1, 2, ..., m), (k = 1, 2, ..., n). \qquad (17)$$

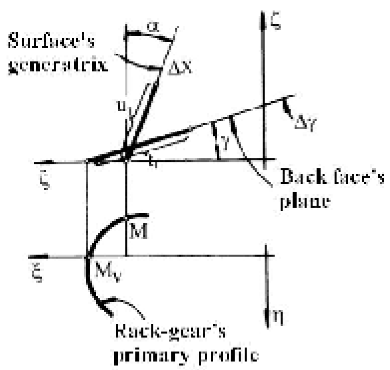

Fig. 5. *Tool's theoretically profile -P_T*

As is know, the totality of points belongs to array (16), and which satisfy, for the same rolling position the condition that the distance (17) to be minimum, represent the corrected tool's profile.

In this case too, the corrected profile, determined form (16) and (17) is an ideal profile.

The effective cutting tool's edge result form the intersection between the back face with the clearance face, see figure 5.

3.1. The approximated profile

The primary (theoretically) profile of the rack-gear tool has an express on form

$$P_T = \begin{Vmatrix} x_{T_1} & x_{T_2} & \mathbf{L} & x_{T_i} & \mathbf{L} & x_{T_n} \\ h_{T_1} & h_{T_2} & \mathbf{L} & h_{T_i} & \mathbf{L} & h_{T_n} \end{Vmatrix}^T . \quad (18)$$

For the current point on the tool's theoretically profile, the clearance surface generatrix family has the equations:

$$(\Delta a) \quad \begin{vmatrix} x = x_{Ti} - u_1 \sin a; \\ h = h_{Ti}; \\ z = u_1 \cos a, \quad (i = 1, 2, ..., n); \end{vmatrix} \quad (19)$$

- [ξ_{Ti}, η_{Ti},] are the current profile coordinates of the primary peripheral tool's surface in ξη plane;
- u_1 is the variable parameter.

The back face of rack-gear is defined as plane which, containing the P_T point with coordinates (ζ_v, η_v), (figure 5), has the γ angle with the ζη plane of the reference system, see :

$$S_g \quad \begin{vmatrix} x = x_v - t_1 \cos g; \\ h = h_v; \\ z = t_1 \sin g. \end{vmatrix} \quad (20)$$

In (20) were defined:
- point with [ζv, ηv] coordinates is the point on the theoretically profile with the maximum value of the ξ, coordinate from (19);
- t_1 - variable parameter.

The intersection between the (Δα) clearance surface generatrix family and the plane of back surface represent the effective tool's cutting edge profile in the ξη plane –P_A, figure 6- approximated profile,

$$P_A: \quad \begin{vmatrix} x = x_v - \dfrac{x_v - x_t}{(\cos a - \sin g\, tga)} \cos g; \\ h = h_t. \end{vmatrix} \quad (21)$$

The (21) equations leader at an express for the approximated profile P_A, in form

$$P_A = \begin{Vmatrix} x_{A_1} & x_{A_2} & \mathbf{L} & x_{A_n} \\ h_{A_1} & h_{A_2} & \mathbf{L} & h_{A_n} \end{Vmatrix}^T . \quad (22)$$

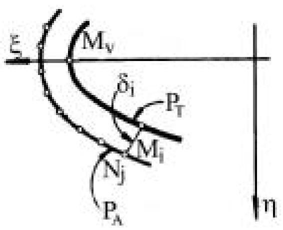

Fig. 6. *Effective profile of cutting edge (approximated profile)*

If the two profiles, the corrected one (P_T) and the approximated one (P_A), are described by enough points, is possible to define a relative position between these two profiles.

Are calculated the distances

$$d_{i,j} = \sqrt{(x_{A_j} - x_{T_i})^2 - (h_{A_j} - h_{T_i})^2}, \quad (23)$$
$$i = 1, 2, ..., n \text{ and } j=1,2,...,m.$$

The minimum distances values from (23) represent, for M_i and N_i, a value comparable with the distance measured on normal at one of the curves between P_T and P_A.

The $\delta_{i,j}$ values are limited at a small enough value to may be accepted that technically the two profiles are identically.

The γ angle value for which P_T and P_A are closed represent the rack-gear tool's back angle with corrected profile.

4. Applications

It's considered a model of the effective surface (fig. 7, AB zone), for which is proposed a model of the virtual generatrix, AC zone, on form:

$$X = -a - u\sin q; \qquad (24)$$
$$Y = u\cos q,$$

which, for u variable, determine a G^V virtual generatrix, with coordinates, for $q = 1^o$, presented in table 1. Based on the surface's virtual generatrix, is profiled the rack-gear tool in conditions:

$a = 10\ mm$ - half of square's flank;

$g = 0^o$ - back angle of the rack-gear tool,

obtaining the corrected tool's profile.

In figures 8 and 9 and in tables 2 and 3, are presented:

- tool's corrected profile ($a = 8^o$; $g = 12^o$), figure 8 and table 2;
- tool's approximated profile ($a = 8^o$; $g = 0^o$), figure 9 and table 3;
- tolerance zone limits of the corrected profile.

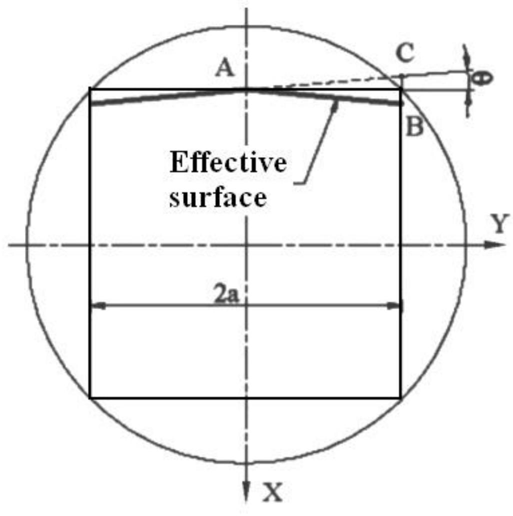

Fig. 7. *Effective surface*

Table 1. *G^V virtual generatrix*

Nr. crt.	X_F [mm]	Y_F [mm]
1	-10.00000	0.00000
2	-10.00017	0.01000
3	-10.00035	0.02000
…	…	…
998	-10.17400	9.96848
999	-10.17418	9.97848
1000	-10.17435	9.98848

The tolerance zone limits for a symmetrical deviation, are modeled with:

$$x = x_e \mp d \cdot \cos b_i; \quad h = h_e \pm d \cdot \sin b_i; \qquad (25)$$

$$b_i = arctg \left| \frac{x_{e_{i+1}} - x_{e_i}}{h_{e_{i+1}} - h_{e_i}} \right| \qquad (26)$$

where d is the allowed deviance value.

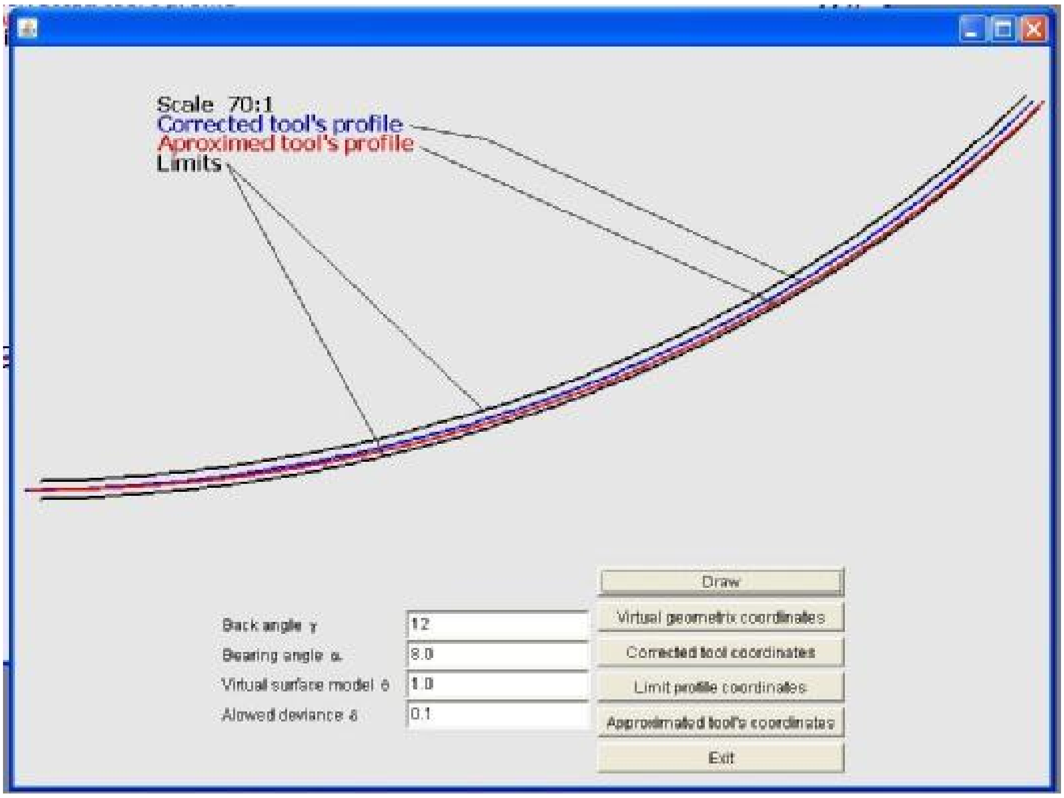

Fig. 8. *Corrected, approximated and limits profiles for rack-gear tool*

Table 2. *Coordinates for a=8° and g=12°*

Crt. no.	Corrected profile		Upper limits		Lower limits		Approximated profile	
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	ξ [mm]	η [mm]	ξ [mm]	η [mm]
0	4,14228	-0,02236	4,03940	0,14682	4,23935	0,15124	4,14213	0,01293
1	4,13937	0,14903	4,03919	0,15634	4,23912	0,16171	4,14212	0,02586
2	4,13915	0,15903	4,03460	0,32714	4,23450	0,33355	4,14209	0,03879
3	4,13455	0,33035	4,03430	0,33664	4,23416	0,34405	4,14206	0,05172
4	4,13423	0,34034	4,02759	0,51735	4,22742	0,52576	4,14202	0,06464
5	4,12750	0,52156	4,02720	0,52682	4,22697	0,53628	4,14197	0,07757
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
151	0,09686	10,76437	0,01861	10,70139	0,16116	10,84168	-0,09110	11,07168
152	0,08988	10,77154	-0,04479	10,76581	0,09711	10,90676	-0,09839	11,07881
153	0,02616	10,83628	-0,05146	10,77253	0,08968	10,91423	-0,10568	11,08593
154	0,01911	10,84338	-0,11478	10,83561	0,02569	10,97797	-0,11297	11,09304
157	-0,11524	10,97587	-0,12152	10,84225	0,01819	10,98537	-0,12026	11,10013

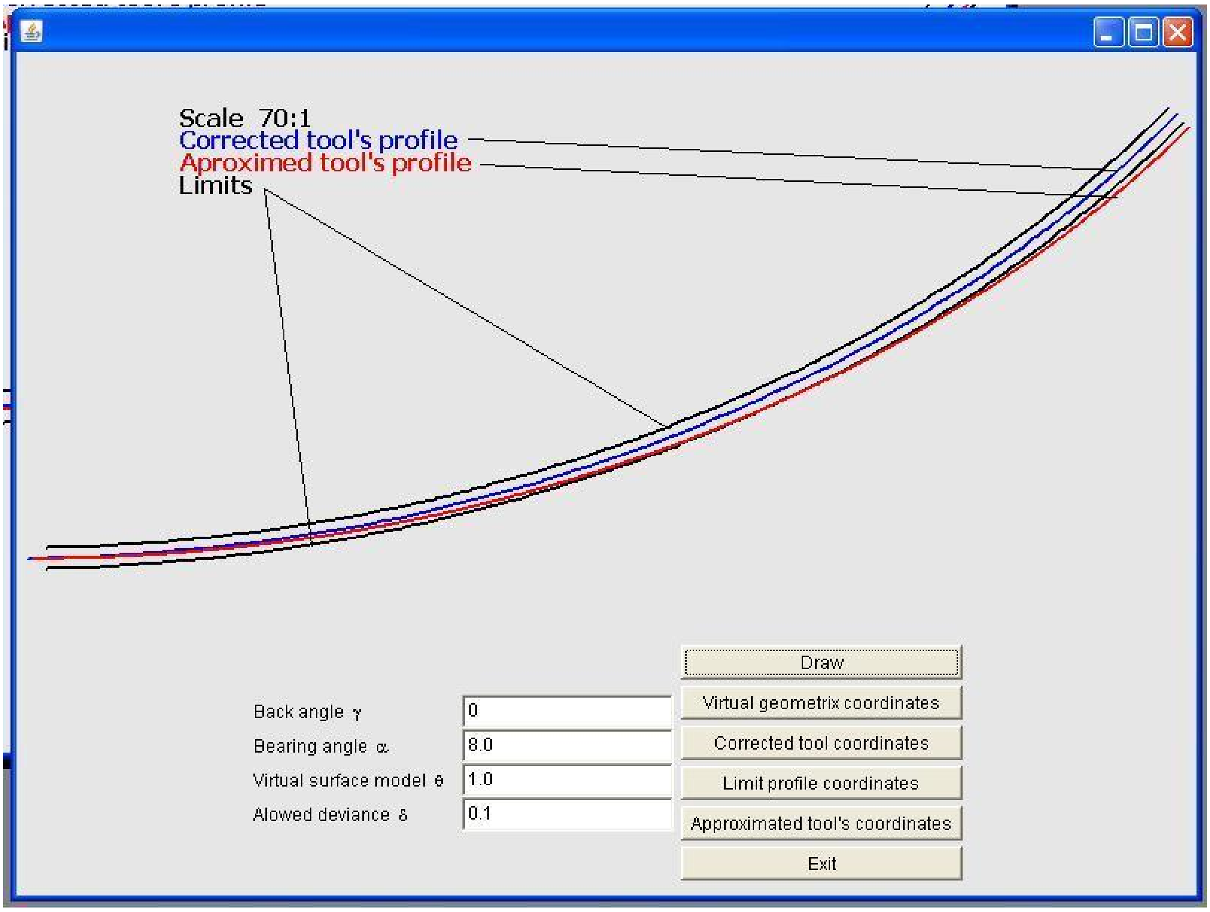

Fig. 9. *Corrected, approximated and limits profiles for rack-gear tool, unacceptable values*

Table 3. *Coordinates for a=8° and g=0°*

Crt. no.	Corrected profile		Upper limits		Lower limits		Approximated profile	
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	ξ [mm]	η [mm]	ξ [mm]	η [mm]
0	4,14228	-0,02236	4,03940	0,14682	4,23935	0,15124	4,14213	0,01293
1	4,13937	0,14903	4,03919	0,15634	4,23912	0,16171	4,14212	0,02586
2	4,13915	0,15903	4,03460	0,32714	4,23450	0,33355	4,14209	0,03879
3	4,13455	0,33035	4,03430	0,33664	4,23416	0,34405	4,14206	0,05172
4	4,13423	0,34034	4,02759	0,51735	4,22742	0,52576	4,14202	0,06464
5	4,12750	0,52156	4,02720	0,52682	4,22697	0,53628	4,14197	0,07757
M	M	M	M	M	M	M	M	M
151	0,09686	10,76437	0,01861	10,70139	0,16116	10,84168	0,05656	11,05019
152	0,08988	10,77154	-0,04479	10,76581	0,09711	10,90676	0,04950	11,05736
153	0,02616	10,83628	-0,05146	10,77253	0,08968	10,91423	0,04242	11,06453
156	-0,05166	10,91381	-0,11478	10,83561	0,02569	10,97797	0,03535	11,07168
157	-0,11524	10,97587	-0,12152	10,84225	0,01819	10,98537	0,02828	11,07881

Note: The proposed method allows correcting the tool’s effective profile. The allowed deviances presented in the applet’s dialog box have values only for the graphical representation of the profiles. Obviously, extremes values of the back angle may lead at unacceptable forms for the tool’s approximated profile.

5. Conclusion

The rack-gear tool’s corrective method for the generation by rolling may assure a tool’s re-profiling by a simple re-sharpening.

The approximated profile may be bringing between the imposed limits for the corrected tool’s profile.

The α and γ geometrical parameters value has influence on the corrected profile's form.

The software is easy to use and allow obtaining a great point's number as so as a graphical representation of the profile.

The method may be used for another tool's type re-profiling.

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Algoritm pentru profilarea corectivă a sculelor pieptene

Rezumat

Se prezintă un model matematic pornind de la o generatoare fictivă a suprafeței de generat şi, în baza acestuia, un algoritm pentru profilarea de corecție a sculelor de tip pieptene, în vederea creşterii preciziei de generare prin înfăşurare cu astfel de scule.

Se acceptă că suprafețele de generat —suprafețele țintă— sunt suprafețe efectiv măsurate şi sunt exprimate în formă numerică. Este imaginat un model de aproximare a profilului corectat al cuțitului pieptene prin modificarea poziției suprafeței de degajare față de forma standard —profilul aproximat.

A fost elaborat un produs soft dedicat acestui model şi sunt prezentate exemple numerice.

Algorithme pour le profilage correctif des outils crémaillère

Résume

Il a été présenté un modèle mathématique, commençant de generatrix virtuel de la surface à être produit et, y basé, un algorithme pour le fait de dresser le portrait correctif d'outil crémaillère, pour augmenter la précision usinant à la génération en roulant de ce type d'outils.

Nous admettons que les surfaces à être produites, les surfaces prévues, sont des surfaces mesurées et sont exprimées dans la forme numérique. Est imaginé un modèle pour l'approximation du profil d'instrument d'égouttoir corrigé par l'arrière forme de visage changeante à propos la forme standard – le profil rapproché.

Il a été élaboré un logiciel pour ce modèle et aussi ont présenté les exemples numériques.