Identification of Sheet Material Parameters from an Inverse Analysis of the Erichsen Test


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ABSTRACT

During the deep drawing process the deformation history is very different as compared to the tensile one where the constitutive equation can be identified only for small values of the plastic strain. More accurate values of the constitutive parameters can be obtained using the drawing experimental device like as the Erichsen one. In this work we propose to use the inverse analysis principle to identify the rheological parameters directly from the Erichsen test. Results obtained for a classical steel will be analyzed using an identification numerical high board (OPTPAR) automatically coupled with a commercial finite element code charged to simulate numerically the experimental test. Application to a sheet aluminum alloy will be presented.

Keywords: Erichsen Test, Inverse Analysis, Finite Element Simulation

1. Introduction

Actually the deep drawing process is one of the most important sheet metal forming process [1]. Owing to the enormous quantity of sheet metal which is used to form pressings (e.g. for motorcar bodies), for many years there has been need of a rapid test which could be used to differentiate the different metals for use in metal forming operations like cold forging, cold extrusion, cold rolling, tube-sinking or wire-drawing. Erichsen test is a very popular test which gives an excellent account of current metallurgical practice in producing pressed and deep drawn components.

In order to understand the whole complexity of the process and to optimize the forming conditions, a numerical modeling must be used [2-5]. This simulation requires available constitutive equations and accurate rheological parameters values which characterize the sheet material behavior. In order to identify the material rheological behavior, the uniaxial tensile test is generally used and the parameters values are obtained approximately by an analytical analysis using same restricted hypothesis: homogeneity of the strain and small influence of the necking area [6]. In this case the stress-strain curve is available only for small plastic deformations. Moreover the problem is that during deep drawing process the deformation history is very different as compared to the uniaxial tensile one and the constitutive equation can be wrong identified.

Generally, for the determination of the sheet metals properties we can use two different experimental tests which as the Olsen and the Erichsen one [7-8]. For the both cases as an indicator of deformability is used the height of the spherical punch until the fracture of the material corresponding to a fissure of a length approximately equal to 5 mm occurs. In several cases is recorded the maximal values of the force corresponding to the necking of the sheet. Accurate values of the constitutive parameters can be obtained by inverse analysis [9-11] directly from the drawing experimental device like as the Erichsen one.

The principal experimental data can be represented by the recordings of the evolution of the drawing force with the depth of the sheet. The identification of the material coefficients is performed with a finite element simulation of the Erichsen test using an optimization procedure. The least squares cost function is expressed in terms of the experimental forces and of the numerical ones.

Numerical results will be presented using an identification high board (named OPTPAR)
automatically coupled with a commercial finite element code (FORGE2® or MARC [12]) charged to simulate numerically the Erichsen test.

2. Experimental set-up

The experimental Erichsen test used in the Europe has a hemispherical punch of steel with the diameter of 20 mm, an active die with the diameter of 27 mm, a restraint plate with the diameter of 33 mm and a sheet with the diameter of 90 mm. The radius of the die is 0.75 mm (see Figure 1).

In the Erichsen test, a hemispherical punch is pressed into the sheet until fracture occurs, at which point the test is stopped immediately and the depth of the bulge measured.

In a pure stretch forming the sheet is totally clamped and is deformed by the punch. The measured depth (mm) gives the Erichsen indices and obviously gives a measure of the ductility of the sheet in the plane of drawing under biaxial stress conditions. Measurement of the axial punch force variation can be added and considered as the experimental data.

3. Numerical modelling and analysis

The experimental device of the Erichsen test can be modeled by the finite element method. We consider a steel sheet with the thickness of 3 mm and diameter of 90 mm. The punch, the restraint plate and the die are considered to be rigid tools and the sheet is meshed using three nodes linear triangular elements (Figure 2).

Remeshing procedure is activated during the process computation in order to eliminate numerical problems linked to the possible appearance of degenerate elements. The contact between the sheet and the restraint plate or the horizontal part of the die is chosen to be glued. Coulomb friction law with a friction coefficient of the 0.1 is used between the sheet and the punch or active part of the die.

Considering the rheological law of the sheet material we choose two different descriptions: a Ludwick one and a Voce one (see Figure 3).

Fig. 1. Experimental set-up of the Erichsen test: a) 3D view (4, 6, 7 – assembly components), b) 2D axisymmetric view.

Fig. 2. Numerical finite element model

Fig. 3. Rheological laws used for the numerical simulation
The rheological equations are defined by:

\[
\bar{\sigma} = \sigma_{00} + K \bar{\varepsilon}^n
\]  (1)

\[
\bar{\sigma} = \sigma_{00} + K\left[1 - \exp\left(-n\bar{\varepsilon}\right)\right]^{n}
\]  (2)

where \(\sigma_{00}\) is the elastic limit of the equivalent Von-Mises stress, \(K\) is the hardening stress and \(n\) is the hardening parameter. For a steel material we have \(n_a = 1\).

The values of the elastic material properties and of the plastic parameters are presented in the Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic</th>
<th>Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(E)</td>
<td>(\nu)</td>
</tr>
<tr>
<td>Steel sheet</td>
<td>210000 MPa</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Plastic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\sigma_{00})</td>
<td>(K)</td>
</tr>
<tr>
<td>Ludwick Law</td>
<td>324 MPa</td>
<td>401.76 MPa</td>
</tr>
<tr>
<td>Voce Law</td>
<td>324 MPa</td>
<td>281.46 MPa</td>
</tr>
</tbody>
</table>

**Fig. 4.** Numerical finite element results with FORGE2 (Mesh and cumulated plastic strain corresponding at \(t = 18\) s)

**Fig. 5.** Numerical finite element results with MARC (cumulated plastic strain at \(t = 18\) s)

Numerical simulation was made with FORGE2 code and the MARC code. Results of the deformed mesh and of the equivalent plastic strain distribution, after 18 mm of the punch displacement, are pictured in Fig. 4 and Fig. 5. The similar results obtained with the both numerical codes confirm the availability of the numerical process description. It is possible to observe the obtained necking phenomena corresponding to the approximately 30 degrees under the punch. Moreover large values of the cumulated plastic strain are obtained (up to 150%) which permits to analyze the deformation history in conditions close to the real stamping process. Results considering the variation of the axial punch force are plotted in Figure 6. These results show that we obtain similar curve shape between the FORGE2 code and the MARC one. The small differences are caused by the numerical treatment of the contact phenomena because different methods are used to regularize numerically the Coulomb law. Concerning the influence of the material rheological law we obtain a more pronounced softening with the Voce stress-strain curve. Used of extrapolation values for the stress from a classical Ludwick law can causes several problems for estimation of the process forces or of the deformation energies.

**Fig. 6.** Results of the axial punch force variation
4. Parameter identification by inverse analysis

Accurate values of the constitutive parameters behavior can be obtained by an inverse analysis. The experimental data can be represented by the recordings of the evolution of the drawing force with the depth of the sheet [Fig. 6]. The identification of the material coefficients is performed with a finite element simulation of the Erichsen test using an optimization procedure (OPTPAR) [Figure 7].

The least squares cost function is expressed in terms of the experimental forces and of the numerical ones in the form:

\[
\Phi(P, F^c, F^{exp}) = \frac{1}{N^{exp}} \sum_{i=1}^{N^{exp}} \left[ F^c_i - F^{exp}_i \right]^2
\]

where \( P \) is the parameter vector: \( P = \{K, n\} \), \( N^{exp} \) is the number of the experimental points, \( F^c \) is the finite element computed punch forces and the \( F^{exp} \) is the corresponding experimental data.

\[ \text{[3]} \]

The high nonlinearity of the objective function requires a robust numerical minimization algorithm based on the Gauss-Newton gradient method. In this case evaluation of the first and second objective function derivatives are required:

\[
\frac{d\Phi}{dP_j} = 2 \sum_{i=1}^{N^{exp}} \left[ \frac{dF^c_i}{dP_j} \right] \left[ F^c_i - F^{exp}_i \right]
\]

\[ \text{[5]} \]

\[
\frac{d^2\Phi}{dP_{jk}} \approx 2 \sum_{i=1}^{N^{exp}} \left[ \frac{dF^c_i}{dP_j} \right] \left[ \frac{dF^c_i}{dP_k} \right]
\]

\[ \text{[6]} \]

A direct differentiation method is used to compute the derivatives of the computed forces with respect to the constitutive parameters. To test the numerical convergences of the inverse analysis procedure, an artificial experimental data based on the constitutive parameter values presented in Table 1, is used.

The goal of convergence analysis is to verify the capacity of the identification algorithm to find the true rheological parameters starting from different initial estimations.

The optimization procedure is started for the both laws using initial values sufficiently far from the real ones. The numerical identification results show that we obtain the numerical convergence in a small number of optimization iterations (10-15 iterations) with a very high precision (objective function smallest that 0.1%). The available numerical inverse analysis model can be now used in a real experimental test.

5. Application to a real experimental data

In order to test the inverse analysis of the Erichsen test in real conditions, rheological parameter identification is made for an aluminum alloy AA5182. The axial punch force is recorded until the fracture of the sheet occurs. After 7.6 mm of the punch displacement the
thickness of the sheet in the necking area is approximately 0.5 mm.

**Table 2.** Numerical parameter identification results for the Ludwick law

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Ludwick Law</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
</tr>
<tr>
<td>$\sigma_{00}$</td>
<td>0.</td>
</tr>
<tr>
<td>$K$</td>
<td>250.</td>
</tr>
<tr>
<td>$n$</td>
<td>2.25</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>72%</td>
</tr>
<tr>
<td>iterations</td>
<td>-</td>
</tr>
</tbody>
</table>

The friction phenomenon and Coulomb parameter it is considered to be a priori known in this study ($\mu = 0.1$). For the Ludwick law the parameter identification results is presented in Table 2.

For the Voce law the numerical identification results is presented in Table 3.

**Table 3.** Numerical parameter identification results for the Voce Law ($n_a=0.5$)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Voce Law ($n_a=0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
</tr>
<tr>
<td>$\sigma_{00}$</td>
<td>0.</td>
</tr>
<tr>
<td>$K$</td>
<td>50.</td>
</tr>
<tr>
<td>$n$</td>
<td>100.</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>45.8%</td>
</tr>
<tr>
<td>iterations</td>
<td>-</td>
</tr>
</tbody>
</table>

These identification results shows that the material have for the Ludwick law a very small hardening effect: $n = 0.01$ and for the Voce law a very high value of the parameter $n$. Moreover the elastic limit is considered to be neglected. All these results permit to conclude that the material behaviour is very close to a rigid plastic one. It is the reason for which a new parameter identification has been made for a rigid plastic law starting from the Ludwick law and considered $K=0$ and $n=0$. The obtained values of $\sigma_{00}$ is approximately 74 MPa.

The comparison between the experimental and the computed loads is plotted in Figure 8. The global error is approximately 16.4% and is principally due to the errors of the experimental recordings caused principally by the glide effect on the restraint plate.

A numerical finite element simulation was made with the code FORGE2 and the deformation of the sheet after 7.6 mm of the punch displacement is pictured in Figure 9. It is important to note that we obtain approximately the same final thickness of the sheet. The cumulated plastic strain in the necking area is more great that 60%.

These results permits to confirm that we have large plastic deformations during the deep drawing process and the use of the classical tensile test for identification of rheological law can be not adequate to describe the forming history of the sheet.

**Figure 8.** Comparison between experimental and computed axial punch forces for a rigid plastic law.

**Figure 9.** Numerical finite element results obtained from FORGE2 (Mesh and cumulated plastic strain corresponding to 7.6 mm of the punch displacement)

In figure 10 is pictured the fracture of the sheet obtained from experiment.

**Figure 10.** Fracture types from experiments
6. Conclusions

The standard and advanced characteristics of the Erichsen test for sheet drawing analysis and also for stamping process optimization have been used to identify the rheological parameters. The finite element model permits to simulate the biaxial stress conditions together with a non-uniform distribution of the plastic strain and of the sheet thickness. It is then possible to obtain close deformations conditions as those, which occur during a real deep drawing process. Numerical results, validations, robustness and integration of inverse analysis in the research codes FORGE 2 and MARC was still performed starting from the time variation of the axial punch force.

7. Acknowledgement

The present project was elaborated until the bilateral research contract between INSA de RENNES - France and UNIVERSITY “Dunarea de Jos” GALATI – Romania and supported in financial terms by the research contracts CEEX 22/2005 and CEEX 24/2006 of the laboratory ITCM of University “Dunarea de Jos” Galati – Romania.

References


Identificarea parametrilor de material ale tablelor utilizând analiza inversă a testului Erichsen

Rezumat

În timpul prelucrării prin ambuiașare adâncă istoria de deformare este foarte diferită în comparație cu cea din testul de tractiune uniaxială unde curba de curgere poate să fie identificată numai pentru valori mici ale deformării plastice. Valori mai corecte ale parametrilor de material pot să fie obținute cu ajutorul dispozitivului experimental de tip Erichsen. În această lucrare propunem utilizarea principiului de analiză inversă pentru a identifica parametrii curbei de curgere utilizând simularea cu elemente finite ale testului Erichsen. Rezultatele obținute pentru un otel clasic demonstrau fiabilitatea programului de identificare numeric (OPTPAR) automat coplat cu un program comercial de simulare numerică. In final este prezentată o aplicatie la studiul comportarii unei table de aluminuim.

Identification des paramètres de matériaux des tôles par analyse inverse du test Erichsen

Résumé

Pendant les processus d’emboutissage profonde l’histoire de déformation plastique est différente en comparaison avec la sollicitation de traction uniaxiale, ou la courbe d’écrouissage peut être identifie seulement pour des valeurs faibles de déformation. Une identification des paramètres plus judicieuse peut être obtenue avec le test Erichsen. Dans cet article on propose l’application du principe d’analyse inverse en utilisant une simulation éléments finis du test expérimental de type Erichsen. Les résultats numériques obtenus pour in acier classique montre la faisabilité du logiciel d’identification des paramètres (OPTPAR) automatiquement couple avec un logiciel commercial de simulation numérique. En final on présente une application pour l’étude du comportement d’une tôle d’aluminium.